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ABSTRACT

This set of papers, selected from presentations at the Fourth and Fifth Annual Conferences of the Research Council for Diagnostic and Prescriptive Mathematics, are of primary interest to mathematics educators. In the first paper, Romberg describes a model for diagnosing mathematical learning difficulties which extends the diagnostic process beyond the mere identification of unpassed objectives. Underhill then provides a basis for distinguishing between learning hierarchies and diagnostic hierarchies, pointing out differences in methods used to validate each. Reports by Hill and Underhill, and by Underhill and Causey explain the methods used to generate diagnostic hierarchies. Novillis discusses the effect of language on learning mathematics, presenting a method for investigating children's acquisition of mathematical terms. Research studies on diagnosing mathematical learning difficulties among deaf children are reviewed by Sinatra, while Sovchik explores the effects of anxiety on mathematical learning. Crenson provides concise descriptions of several tests for assessing students' perceptual skills and critiques the role of learning style in the diagnostic process. Finally, Fennell reports on a study comparing two approaches for training preservice teachers to use diagnostic-prescriptive methods. (MNS)

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INTRODUCTION

Papers presented at the Fourth and Fifth Annual Conferences of RCDPM covered a wide variety of topics ranging in content from theoretical research models for developing diagnostic models to ideas for implementing diagnostic-prescriptive techniques in the classroom. The spectrum of topics included papers which would be of primary interest to mathematics educators. These papers were selected for publication in this monograph.

In the first paper of this monograph, Romberg describes a model for diagnosing mathematical learning difficulties which extends the diagnostic process beyond the mere identification of objectives not passed. The next article, by Underhill, provides a basis for distinguishing between learning hierarchies and diagnostic hierarchies by pointing out the differences in the methods used to validate each type of hierarchy. Following Underhill's paper are reports on two studies, Hill and Causey, which explain the methods used in generating diagnostic hierarchies. Hill's report provides a hierarchy for long multiplication of rational numbers.

The papers by Novillis, Sinatra, Sovchik and Crenson provide reviews of research studies which investigated the affects various variables have on the learning of mathematics. Novillis' paper discusses the affects of language on learning mathematics, and presents a method for investigating children's acquisition of mathematical terms. Research studies concerned with the diagnosis of mathematical learning difficulties among deaf children are reviewed by Sinatra. The affects of anxiety on mathematical learning are explored in the paper by Sovchik. The article by Crenson provides concise descriptions of several tests for assessing students' preceptual skills and a thought provoking critique of the importance of a child's learning style in the diagnostic process.

In the final paper of this monograph, Fennell reports on a study which compared two approaches for training preservice teachers in the use of diagnostic-prescriptive methods.

The RCDPM Publications Committee wishes to acknowledge the author's contributions to RCDPM and to the profession through their willingness to share their work with the readers of this monograph. The Committee would also like to express their appreciation to Mrs. Lucy Harward, graduate student at Florida State University, for her editorial assistance in preparing this monograph for publication.

Tom Denmark
Editor

SECOND-LEVEL DIAGNOSIS: ASSESSING UNDERLYING PROCESSES, NOT JUST OBJECTIVES

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The purpose of this paper is to present a framework for a *second-level* diagnosis which goes beyond the identification of discrepancies with respect to performance on specific instructional objectives. That is, usually the term is used based on an assumption that a student has failed to perform with respect to some particular objective. (There is a difference between actual performance and expected performance on a particular class of problems.) What I am claiming is that, given discrepancies identified at the *first-level*, a *second-level* of diagnosis is then called for.

First-Level Diagnosis. To carry out an initial *first-level* diagnosis, instructional objectives must be carefully stated and assessed. An instructional objective has five components (Romberg, 1976). These are: antecedent, subject, verb, consequent and criterion. The antecedent is a description of a situation in which students are to work. The stimulus resources and/or immediate environment may be described. The subject of an instructional objective designates the individual or individuals referred to in the antecedent. The verb is the descriptor of the mode in which the objective is executed. The mode can reflect a general or behavioral orientation. The consequent is a descriptor of the anticipated response. It is the terminal evidence of the antecedent conditions. The criterion is the level of proficiency the student is to attain that is the standard on which a decision will be made as to whether the student has attained the objectives. In summary, statements in the following form are instructional objectives: "The subject when exposed to the conditions prescribed in the antecedent displays the actions specified in the verb in a situation specified by the consequence to some specified criteria." When objectives are stated in this manner, a sample of situations can be created and responses observed and judged. A *first-level* diagnosis is based on a stimulus-response relationship (see Figure 1). The stimulus is presented to the subject, and the subject responds in one or more ways--an affective response, a cognitive response, a behavioral response. The response is then examined and judged.

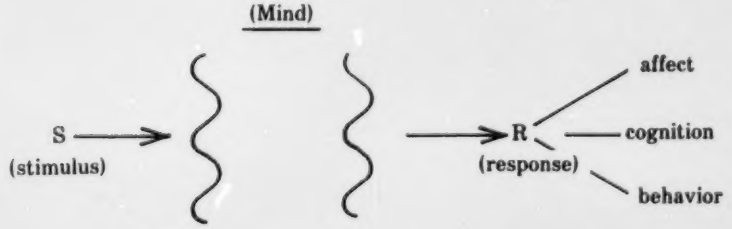

Figure 1

The stimulus-response relationship implied by instructional objectives.

Second-Level Diagnosis

If the response is judged to be discrepant from some expectation, one has diagnosed the symptoms of an underlying problem. But knowing symptoms is not the same as knowing probable causes of that problem. *First-level* diagnosis is analogous to taking one's temperature or blood pressure. Such information gives indications of some discrepancy in bodily functions but does not give medical practitioners specific information on how to remediate the illness.

Second-Level Diagnosis. The second step I am arguing for must involve an attempt to study what happens between S and R in Figure 1. This attempt is what I am calling *second-level* diagnosis. Not being a psychologist, I have found the following paradigm (Figure 2) convenient (Romberg, 1969). Solving a problem involves the perception of the stimulus situation (identification of the problem); the establishment of some overall plan, procedure, process, or strategy in order to answer the particular problem (selection of a general intellectual process to be used to solve this problem); the coding of the information from the stimulus situation (identifying the dimensions upon which the problem is to be solved and noting the relative values of objects in that dimension or those dimensions); memory (storage and/or retrieval) of the coded data; and imposing transformations (formal algorithms or mediational elaborations) upon the encoded data.

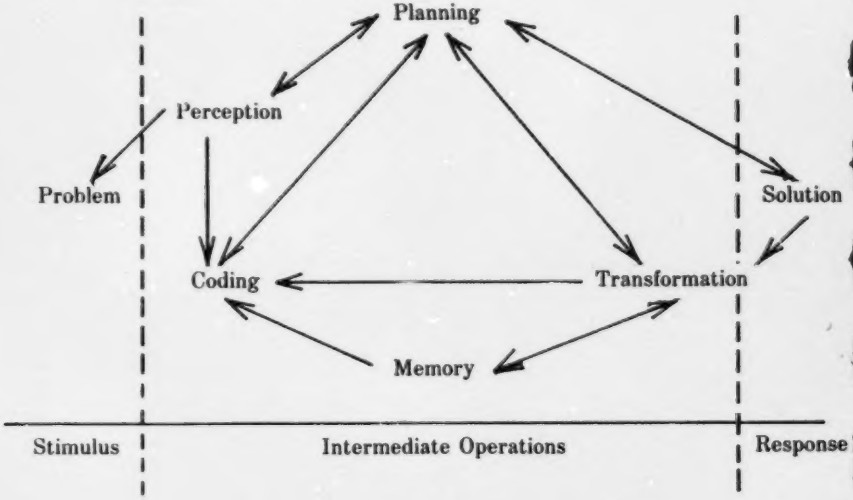

Figure 2

Types of Cognitive Operations (Romberg, 1969).

What I am arguing is that *second-level* diagnosis involves identifying which intermediate operation (or operations) has failed.

Perceptual Processes. In many cases it can be argued that the message presented in the stimulus is not understood or perceived correctly by the students. For example, Romberg and Scott (1968) when measuring performance on conservation tasks found that there was a dramatic increase in correct responses when the instructions to each question were stated twice. Also, in a remediation study (Heck, Maalouf & Romberg, 1975) with students who had not acquired skills to do a two-digit algorithm for adding whole numbers with regrouping, repeating the instructions and pointing out the steps that they should be attending to in the particular task was a successful procedure for both increasing performance and transferring to a three-digit algorithm. I might add that in that study we hypothesized that studying the operation from a different perspective would help. The repeated instruction method was conceived as a control. We had assumed the difficulty was because of a coding failure to see relationship between the algorithm and an underlying conceptual basis.

The message is in the receiver not in the sender. If the message of a stimulus situation is not perceived accurately by students, they are not likely to produce a desired response. My guess is that in instructional situations inaccurate perception is a more prevalent common failure than is commonly thought.

Planning Processes. The second set of processes associated with cognitive functioning is planning processes. The literature in cognitive psychology indicates that human beings differ in the way in which they organize and manage information in a particular problem situation. For example, Piaget's stage notion can be interpreted in terms of stages when children are able to organize and handle information. In several studies at the Wisconsin Research and Development Center, we tried to get young students to handle problems at a relatively abstract hypothetical level (formal operations). We have been universally unsuccessful. For example, Shepler (1969) taught students several probability tasks. He was very successful in getting students to be able to do empirical probability problems, but was unable to get students to handle abstract problems. Lovell (1971) pointed out that Shepler was asking students to operate at Piaget's formal operational stage. They were not able to manage the abstract information. For young children, variation in planning operations may be due to developmental maturity rather than instructional experience. This is a hypothesis that needs to be examined. However, there have been several studies at the collegiate level where it has been demonstrated that planning heuristics such as those suggested by George Polya (1957) can indeed be learned (for example, see Lucas, 1972).

Coding Processes. The next category of cognitive functioning involves the neurological coding processes within the brain. It seems unlikely that this is something that classroom teachers can be concerned with for most students. However, there is a line of research that has looked at this particular kind of malfunction. If it referred to technically as "acalculia." For the interested reader a good introductory reference is Grewel (1973).

Memory Processes. The fourth category of processes where cognitive functioning may break down is memory. All humans are continually bombarded with information which the human mind is unable to store and utilize. Thus, most of the information that is processed at one time or another by any human being is discarded and not kept in memory. Questions of what and how much information is retained in memory are critical (in particular see Case, 1978). It is common that particular routines, concepts, and skills, unless reinforced, are discarded. Only by tying new ideas with the existing cognitive structure will information be retained. I assume most mathematics educators have had a course in calculus, but now most would be unable to integrate a particular expression. That does not mean that one would need to be taught integral calculus again. Given a few minutes with a text and a few examples, it would not take long for most of us to do that particular task. The difference between never having had the opportunity to learn a set of skills and not remembering how to use them is a critical distinction.

Transformation Processes. I think the most important set of cognitive processes which math teachers need to be aware of are the transformation skills. Elsewhere (Romberg, 1975) I have written extensively on this particular topic since this is the primary concern in my own research.

Transformation operations are the *intellectual skills* which solve problems. Any suggestion that these skills are well understood would be an oversimplification. What we do know is that for mathematical problem solving a complex set of interrelated skills are used. Some students learn the skills easily, almost as a natural development; other students experience considerable difficulty in learning one or more of the skills.

We also know that there are some skills children can use actively to solve problems in their world. When children enter school, their reality is concrete and includes mainly the objects, people and situations around them or those made real to them through pictures and stories. They have little in common with the abstract and symbolic world of mathematics of adults. It should be obvious that is students are unable to solve certain problems, one reason could be that particular skills have not been mastered.

The procedure one should use for *second-level* diagnosis is known as *sequential* or *tailored testing* (Cronbach & Glaser, 1965). Given information from a *first-level* diagnosis, a sequential testing program aimed at gathering specific data on underlying characteristics is then tailored to the individual. This procedure is well known in the psychometric literature but practical examples are rare. *Second-level* diagnosis always involves individual assessment.

In summary, *second-level* diagnosis goes beyond the identification of a discrepancy between actual and intended performance. It attempts to find where within the types of cognitive operations the mental system has broken down. The most likely sources of breakdown are whether in the perceptual operations--failure to understand the problem or in the capability of using a particular transformational operation. In either case, it can be assumed that both these cognitive operation deficiencies can be overcome through instruction.

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SOME THEORETICAL AND RESEARCH ISSUES RELATED TO WORK WITH DIAGNOSTIC HIERARCHIES

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Several researchers have used alternate strategies for generating and validating a certain class of hierarchies, but a basic theoretical issue is involved. Simply stated, what is the relation between learning hierarchies and subskill retention hierarchies?

Learning hierarchies reflect optimal sequences for original learning: hence, they are of great importance in planning sequences for curriculum content not previously encountered by learners. Optimal paths for original learning certainly represent a major concern of teachers and curriculum planners.

Of equal concern is the planning of curriculum content sequences for learners who have traversed a learning sequence one or more times but who cannot successfully demonstrate the skills and subskills presented in the instructional sequence(s). These latter sequences, while they may be quite different from learning hierarchies, are very important in the development of diagnostic instruments and prescriptive strategies.

Learning Hierarchies

Primarily as a result of Gagne's conceptualization of task analysis, several studies were completed in the 'sixties based on epistemological considerations of hierarchical structures of knowledge (Gagne and Paradise, 1961; Gagne, 1962; Cox and Graham, 1963; Uprichard, 1970). While the studies supported the existence of hierarchies, Gagne (1968) and others (Pyatte, 1969; White, 1974a) point out the difficulties in validating the hierarchies.

Several statistical techniques for hierarchy validation exist. These have been reviewed and critiques in several sources (Resnich and Wang, 1969; Eisenberg and Walbesser, 1971; White, 1973, 1974a, 1974b; Phillips, 1971, 1974). The procedures have been presented in terms of learning hierarchies. That is, the studies reported have been primarily concerned with the development of optimal sequences for original learning. White (1974b) summarized and critiqued several statistical techniques for validating task dependencies among subtasks of a superordinate task. He concluded that learning hierarchies can best be established through research on instructional sequences created by forming all possible permutations of subtasks. Instruction, data collection, and analysis of all possible permutations of subtasks in a hierarchy is a procedure which expends enormous quantities of resources. Furthermore, the relation between optimal original learning sequences and diagnostic/remedial sequences is not known.

Subskill Retention Hierarchies

Uprichard and Phillips sought to develop a strategy which would yield helpful information within the framework of lower resource expenditure and which might yield more valid information for diagnostic work. They completed two investigations on addition (1977) and subtraction (1976) of fractions. In the addition study, they generated 89 specific subtasks which were subsequently collapsed into 45 subtasks. A test was constructed and administered to 251 subjects in grades four through eight. In the subtraction study, they generated

96 subtasks which were subsequently collapsed into 47 subtasks. Similarly, a test was constructed and administered to 100 subjects in grades four through eight.

The significant issue related to their investigation is now more clear: What is the relation between learning hierarchies proposed by Gagne and others and the hierarchies hypothesized by Uprichard and Phillips?

The Issue

Gagne defined a learning hierarchy in terms of original learning. While numerous hierarchies may exist, an optimal learning hierarchy is one through which a learner can pass most efficiently with understanding and mastery. While Uprichard and Phillips have interest and concerns related to learning hierarchies, two matters inhibit them from making definitive conclusions about optimal learning hierarchies as direct outcomes of their investigations.

First, the subjects used in the Uprichard and Phillips studies were progressing through a relatively small number of permuted hierarchical combinations as reflected in their school curricula, so the resulting hierarchical dependencies may reflect the optimal sequences only among learning hierarchies through which these subjects progressed. Thus, their research results are of very limited value in relation to optimal learning sequences. A further complication results from Gagne and Bassler's (1963) conclusion in a study wherein subjects who learned a superordinate task were found to no longer retain some supraordinate tasks which were known to be prerequisite. From this latter study, it follows that those students who could demonstrate higher order subtasks on Uprichard and Phillips' instruments may have learned but subsequently forgot certain subtasks.

Second, the use of data from a broad cross section of students in grades four through eight to study addition of fractions is a plausible alternative, but how are the resulting hierarchical dependencies related to learning hierarchies? The hierarchies resulting from the Uprichard and Phillips analyses of ex post facto data relative to many of the supraordinate and, in some cases, the superordinate task reflects something other than a learning hierarchy. These may better be called subskill retention hierarchies. They are subskill retention structures or mappings of retained subordinate task capabilities. The relation between these subskill retention hierarchies and learning hierarchies is not known, either theoretically or empirically. The general nature of hierarchies which result from analyzing data collected from subjects who have encountered instruction in the task and subtasks one or more times is not understood. Fourth graders in the Uprichard and Phillips studies had encountered some of the subtasks related to addition and subtraction of fractions. Their eighth graders had probably encountered all of those subtasks and the superordinate task one or more times.

Although Uprichard and Phillips claimed to be generating learning hierarchies, there is empirical evidence upon which to question this claim. Data analyses in all four of the completed intraconcept analysis studies (Uprichard and Phillips, 1976, 1977; Hill, 1977; Causey, 1976) have been collected from subjects who have already been taught how to add, subtract, multiply and divide. Thus, there are significant factors related to retention, original

learning, overlearning, and interference which probably influenced children's performances. Further, the research by Gagne and Bassler cited previously indicates that in some instances learning prerequisites may not be retained once superordinate tasks are learned, i.e., if x is a prerequisite to y, x may be forgotten one y is learned. Thus, once children know how to add mixed numerals, they may no longer be able to perform some subtasks necessary for learning this superordinate task. While this appears to be a minor factor in mathematics learning, it cannot be ignored.

The hierarchies developed in the four studies are more appropriately referred to as subskill retention hierarchies. That is, after having been taught how to add fractions one or more times, the resulting hierarchy represents the capabilities retained. Such structures hold considerable potential for diagnostic and remedial work; valid patterns of retained subskills may in planning remedial procedures.

Some Procedural and Statistical Issues

While at Purdue, Phillips explored various learning hierarchy validation techniques. Phillips and Kane (1973) concluded that the AAAS (American Academy for the Advancement of Science) procedures developed by Eisenberg and Walbesser (1971) and pattern analysis procedures developed by Rimoldi and Grib (1960) were among the best. Uprichard and Phillips decided to use these two statistical procedures to analyze their data.

Some knowledge of the matters described in the previous section led Phillips (1971) to alter the criterion levels of the AAAS ratios. The AAAS or Walbesser technique is based on a 2 x 2 contigency table of responses based on correct and incorrect responses of vertically positioned elements in a proposed hierarchy (see Figure 1). The consistency ratio is a measure of how consistent the data are with the hypothesized dependency; the adequacy ratio is a measure of the adequacy of the identified hierarchical levels; and the completeness ratio is a measure of the effectiveness of instruction. Phillips modified the criterion levels because he claimed that no instruction was involved. It is more appropriately stated that no immediate instruction was involved on the precise hierarchy hypothesized.

In the two studies completed by Uprichard and Phillips, they hypothesized linear hierarchies and tested the hierarchical relationships with the Walbesser ratios and pattern analysis. Following these analyses, they adjusted the hypothesized hierarchy to their own satisfaction of logical consistency based on task analysis, and they they demonstrated that the changes strengthened the ratios and pattern analysis values. This procedure of modifying the hypothesized hierarchy based on the analysis is not problematic. What is problematic is the lack of cross-validation steps following these modifications.

In the addition study, the "best" linear ordering based on the procedures outlined in the previous paragraph did not yield three acceptable ratio values for each of the 44 cases. The criterion level of the consistency ratio was met in 35 of 44 cases; the criterion level of the adequacy ratio was met in 43 of 44 cases; and the criterion level of the completeness ratio was met in 27 of 44 cases. The pattern analysis index of agreement based on the hypothesized hierarchy was .68. After the tasks were reordered, the index of agreement

was .70. No criterion levels exists for this statistic.

In the subtraction study, the "best" ordering based on the same procedures yielded the following results: The criterion level of the consistency ratio was met in 34 of 46 cases; the criterion level of the adequacy ratio was met in 45 of 46 cases; and the criterion level of the completeness ratio was met in 43 of 46 cases. The pattern analysis index of agreement was .63 on the original hierarchy. When the elements of the hierarchy were rearranged, the index of agreement was .75. As in their earlier study, no cross-validation steps were reported.

		Item n	
		Fail (-)	Pass (+)
Item n + 1	pass (+)	- +	+ +
	fail (-)	- -	+ -

Ratio	Value	Walbesser Criterion	Phillips Criterion
Consistency	$\frac{n(++)}{n(++)+n(+-)}$	.85	.85
Adequacy	$\frac{n(++)}{n(++)+n(-+)}$	.85	.70
Completeness	$\frac{n(++)}{n(++)+n(--)}$	.85	.50

Figure 1.
AAAS Ratios' with Modified Criterion Levels

Summary

One of the problems related to hierarchy development is the nebulousness or ambiguity involved in the narrow ranges of logical analysis. Uprichard and Phillips clearly demonstrated that their original logical analysis could be improved upon by following this sequence:

Hypothesize ⟶ Analyze ⟶ Rehypothesize ⟶ Reanalyze

There are at least three problems involved in this sequence. First, only one set of data were used. This leads to questions of cross-validation. The second problem is that these analyses were based on performances on a particular set of items and may reflect only a unique hierarchical arrangement

among these items. And third, only linear sequences were examined; perhaps this is why criterion levels were not attained in all cases.

Because of the procedures followed, that is, use of a limited set of problems and re-use of data, one might more approriately refer to the Uprichard and Phillips studies as hierarchy generation studies. It seems a bit overstated to say that the hierarchies have been validated.

An Alternative Sequence

In her hierarchy research, Hill (1976) used the Uprichard and Phillips studies as a starting point. However, since she was uncertain of the specific hierarchical relationships among the identified subtasks, she incorporated into her study certain key elements which allowed her to rely more heavily on mean performances on subtasks than had been done previously. Three measures taken by her were: (1) use of a pre-test of prerequisite skills, (2) control of basic fact difficulty, and (3) use of four alternate forms of the data collection instrument. She used the following sequence:

Analyze ⟶ Hypothesize ⟶ Reanalyze

She used the subskill item difficulty indices of over 700 subjects in grades 5, 7, and 9 to hypothesize a linear hierarchy. First, the subskills were rank ordered by difficulty levels, Walbesser ratios were calculated, and pattern analysis was executed. Then, through logical analyses which were consistent with task analysis, a modified ordering was achieved. The Walbesser ratios supported this step and the pattern analysis index of agreement decreased by .05.

In the Hill study, Phillips' recommended criterion levels were used on the Walbesser ratios. The criterion value of the consistency ratio was met in 27 of 29 cases; the criterion value of the adequacy ratio was met in 28 of 29 cases; and the criterion value of the completeness ratio was met in 29 of 29 cases. The pattern analysis index of agreement on the first ordering was .58. When the hierarchical elements were rearranged by task analysis, the new index was .53. The identified sequence appears to have merit. By introducing certain controls at the beginning, it may be possible to eliminate the first level of hypothesizing. A brief summary of Hill's study is presented at the end of this paper.

Branched Hierarchies

The sequence used by Hill was also used by Causey (1976). He used the Uprichard and Phillips strategy to generate 34 subskills related to multiplication of fractions. As in Hill's strategy, Causey used the subskill difficulty levels and logical analysis to generate a linear hierarchy. He administered his instrument to 225 subjects in grades 5, 6, and 7. The analyses did not support the existence of the linear hierarchy hypothesized. A new, branched hierarchy was generated. The branches were subsequently evaluated with Walbesser ratios. Causey utilized the following five steps:

(1) Subskills were linearly ordered by item difficulty.
(2) Subskills were linearly reordered using task analysis.
(3) The linear ordering was tested with the Walbesser ratios.
(4) Linearity was rejected and branches were hypothesized using item difficulty and task analysis.
(5) Branches were examined with Walbesser ratios.

In step (3), 17 subskills met the consistency criterion; 28 met adequacy criterion; and 16 met the completeness criterion. In step (5), 33 met consistency criterion; 21 met adequacy criterion; and 29 met completeness criterion. These substantial improvements, especially in the number of pairings meeting the consistency criterion, indicate that the controls and sequence used by Hill and Causey offer exciting possibilities for the generation of branched hierarchies.

Summary

The four studies cited clearly indicate that researchers are developing alternative procedures. There is a need to standardize alternatives, to eliminate some of the variability, and to strengthen communication language by isolating areas of agreement and disagreement.

Since researchers in this area have used alternative procedures, they need to resolve critical questions of theoretical and statistical importance. A strong need exists to develop statistical and procedural alternatives which accommodate theoretical concerns being generated. The data collected on children who have previously encountered supraordinate and superordinate tasks in their curriculum experiences reflect diagnostic needs relative to basic computational skills. If researchers can develop one or two common approaches, they will have at their disposal valuable tools for future investigative work, and practitioners will have hierarchies for continued explorations in diagnosticprescriptive work in clinical and classroom settings.

The Nature of the Task

From the four studies discussed, four major problem areas arise. These may be stated in question form as follows:

A. What alternative criterion levels of Walbesser ratios can be used when they are applied to the analysis of subskill retention hierarchy data?
B. What alternative criterion levels of the pattern analysis index can be applied to subskill retention hierarchy data?
C. What alternative strategies can be used for generating hierarchies?
D. What are the theoretical and practical relations between learning hierarchies and subskill retention hierarchies?

It is the best judgement of this investigator that researchers can best approach resolution of these questions through a three step process:

(1) Acquaint concerned researchers with the nature of the problems in this area.
(2) Discuss the problems and develop alternate strategies for resolving the issue.
(3) Use existing data to examine and cross-validate the alternatives developed.

AN EMPIRICALLY DERIVED HIERARCHY OF INTRA-CONCEPT RELATIONSHIPS: WHOLE NUMBER DIVISION

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Objective

One purpose of this investigation was to use task analysis, intraconcept analysis, and indirect validation procedures to generate a hierarchy of mathematical relationships related to the traditional long division algorithm. This study included division with a one digit whole number divisor and a whole number dividend with at most three digits. The second purpose was to investigate an alternative research sequence for generating linear hierarchies.

Rationale

Examinations of content structures have been executed by at least two analytical procedures. One is the widely used task analysis procedure, based on the writings of Gagne (1962), which relates the logical organization of tasks related to a given concept only to content factors. Another procedure is the intraconcept analysis procedure, developed by Uprichard and Phillips (1976, 1977), which consideres both psychological and content factors in developing and hierarchically ordering tasks related to a given concept. Both intraconcept analysis and task analysis should facilitate organization of subject content in a hierarchical manner by partitioning each skill into subordinate subskills. Using the techniques of indirect validation a post hoc analysis was used to create a hierarchy for division of whole numbers which does not involve an instructional sequence. This study used numerical procedures developed by Walbesser (1968) and Rimoldi and Grib (1960) to test a hypothesis of task dependency in a hierarchy of mathematical skills.

Uprichard and Phillips used the following sequence:
hypothesize ⟶ analyze ⟶ hypothesize ⟶ reanalyze. In the present study an initial analysis was completed before hypothesis were formulated. By controlling task uniqueness, subject possession of prerequisites, and controlling for difficulty of task elements through use of only easy and moderately difficult basic facts, information from an initial analysis was used to eliminate one step as follows:

analyze ⟶ hypothesize ⟶ reanalyze.

Sampling

A sample of 468 fifth, seventh and ninth grade students of the Houston Independent School District was used.

Instrumentation

An instrument with K-R 21 reliability .95 consisting of 30 tasks based on intraconcept analysis was constructed. The instrument was constructed from the analysis of two classification types. These were dividend-subtraction types and quotient-remainder types. Dividend-subtraction types were defined on the basis of number of digits in the dividend, placement of zeroes in the dividend, and presence of remainder upon first, second, or third divisions involving the divisor and a partial dividend. Quotient-remainder types were defined on the basis of number of digits in a quotient, position of zeroes in the quotient, and existence of a final remainder. Types involving final remainders were not

included in either classification. This analysis yielded twenty-nine tasks. One additional task involving a two digit divisor was included.

Analysis

The analysis included the results from only those subjects who did not answer all tasks correctly or all tasks incorrectly. Item difficulty yielded a linear ordering of the tasks based on the proportion of students answering each task correctly. From this ordering a nesting pattern was discerned based on nine factors with a progressive nesting of the variables in the following order:

1. Number of digits in the dividend
2. Uneven divisions in at least one partial division of quotient
3. Number of uneven divisions
4. Zeroes in the quotient
5. Comparison of number of digits in the dividend with number of digits in the quotient
6. Place value involving uneven divisions
7. Existence of zeroes in the dividend
8. Correspondence of zeroes in the dividend with zeroes in the quotient
9. Position of zeroes in the dividend

Following the use of item difficulty and task analysis, a linear hierarchy was hypothesized, and the following Walbesser ratios were calculated: consistency ratio, adequacy ratio, and completeness ratio. They were calculated for each of the linear relationships proposed in the hypothesized hierarchy and tested at the criterion levels: .85 for consistency, .70 for adequacy and .50 for completeness. The consistency ratio was met in 27 of 29 comparisons, the adequacy ratio was met in 28 of 29, and the completeness ratio in 29 of 29. The results supported the existence of a hierarchical ordering.

The pattern of responses obtained from the derived ordering of subordinate tasks yielded an index of agreement of .53. The empirical sequence yielded an index agreement of .58. No statistical test of significance for the Walbesser ratios or for the index of agreement has been developed.

Results

The following questions answered and nested in order from least difficult to most difficult indicated the relative difficulty of the tasks tested.

QUESTION I. How many digits are in the dividend?
ORDER: one digit, two digits, three digits

QUESTION II. Are there uneven divisions?
ORDER: no uneven division, uneven divisions

QUESTION III. How many uneven division are there?
ORDER: one, two

QUESTION IV. Are there any zeroes in the quotient?
ORDER: no zeroes, zeroes

QUESTION V. Are there the same number of digits in the quotient and dividend?
ORDER: same, different

QUESTION VI. Where are the uneven divisions?
ORDER: tens, hundreds, tens and hundreds

QUESTION VII. Are there zeroes in the dividend?
ORDER: no zeroes, zeroes

QUESTION VIII. Do the zeroes in the dividend correspond in position to zeroes in the quotient?
ORDER: yes, no

QUESTION IX. Where are the zeroes in the dividend?
ORDER: ones, ones and tens, tens

Significance of the Findings

The results of the study support the idea that intraconcept analysis involving psychological and content factors be considered when developing possible remedial strategies, and diagnosing learning difficulties. The results also yield a possible instructional sequence to be presented to students. And, finally, they suggest that the number of steps in linear hierarchy generation can be reduced from four to three.

AN EMPIRICALLY DERIVED HIERARCHY OF INTRA-CONCEPT RELATIONSHIPS: RATIONAL NUMBER MULTIPLICATION

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Objective

One purpose of this investigation was to generate a hierarchy of subskills related to multiplication of rational numbers by using task analysis and intra-concept analysis. These are techniques developed by Gagne (1962) and Uprichard and Phillips (1976, 1977), respectively. The second purpose was to explore the feasibility of generating a branched hierarchy.

Theoretical Background

According to Gagne's theory, knowledge may be organized as hierarchical structures. Structures can be validated in several ways. White (1973) reviewed several validation techniques for learning hierarchies and pointed out pitfalls inherent in post hoc analyses. While the best methods of learning hierarchy validation are through experimental treatments involving permutations of subskills, other methods have been sought which will help establish hierarchies for diagnostic and remedial work. For this context, post hoc analyses offer an alternative for creating hierarchies which may be useful in non-original-instructional settings.

Uprichard and Phillips (1976, 1977) reported two intraconcept analysis studies related to addition and subtraction of rational numbers. They determined that post hoc analyses of children's performances when studied on psychological as well as logical criteria do, indeed, yield useful and largely predictable relationships. In this study, post hoc analyses were used to create branched hierarchy for multiplication of rational numbers.

Sampling

After a piloting of the alternative tests on 254 subjects in grades 5 and 6, the investigators analyzed the performances with Walbesser ratios and decided that a linear ordering of the subtasks would not yield satisfactory ratios. Then 225 subjects in grades 5, 6, and 7 were examined. A test of basic facts was administered and only those who achieved at a criterion-level of 75% on a basic facts test were included in the analysis. Further, only basic facts of easy and moderate difficulty were used in the subtasks.

Instrumentation

Two instruments were developed: (1) Test of Prerequisites, and (2) Test of Rational Number Multiplication. Research findings (Murray, 1939) relative to the difficulty levels of various whole number multiplicaion and division combinations were used in the construction of these instruments.

The first test contained three subtests: 8 multiplication combinations, 8 improper fractions, and 4 mixed numerals. Criterion was 75% overall and 50% on each subtest for the inclusion of a given subject's responses in the hierarchy analyses.

The second test contained thirty-four item types resulting from twenty-four permutations of five number types (integers, unit fractions, non-unit fractions, improper fractions, mixed numerals). Five forms were utilized, The KR-20 was .93.

Rational Number Multiplication

Analysis

The following steps were used to develop the hierarchy:

(1) Tasks were ranked according to difficulty.

(2) Tasks were linearly reordered by intraconcept analysis procedures.

(3) The linear ordering was tested with Walbesser ratios.

(4) Linearity was rejected and branches hypothesized.

(5) Branched hierarchies were examined with Walbesser ratios.

Following the argument of Uprichard and Phillips (1976, 1977), the following Walbesser ratio criterion levels were used: .85 for consistency, .70 for adequacy, and .50 for completeness.

Results

Of 225 subjects tested, 61 failed to meet criterion levels on the Test of Prerequisites; 35 others were eliminated because of incomplete data; 1 was eliminated because all items were correct; and 1 was omitted because all items were incorrect. The resulting sample for the hierarchy analyses was 127.

In step (3) of the analysis only 17 subskills met the consistency criterion level; 28 met adequacy criterion; and 16 met completeness criterion.

In step (5) 33 met the consistency criterion; 21 met the adequacy criterion; and 29 met the completeness criterion.

The following statements reflect the structure of the final branched hierarchy:

(1) Presence of a mixed numeral factor

(2) Presence of a mixed numeral product

(3) Mixed numeral right-most factor

(4) Right-most mixed numeral with integer (least difficult) or improper fraction, or non-unit fraction or unit fraction as second factor.

(5) Left-most mixed numeral with integer (least difficult) or improper fraction, or non-unit fraction or unit fraction as second factor.

Significance of the Findings

This study once again demonstrates that hierarchies can be generated by using task analysis, intraconcept analysis, and Walbesser techniques. Further, this is the first study which has attempted to use these techniques for the generation of branched hierarchies. As such, it opens new questions for exploration relative to the application of these strategies and their usefulness in examining studies already completed which forced data into linear hierarchies.

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A SELECTED REVIEW OF SOME PSYCHOLINGUISTIC STUDIES OF INTEREST TO MATHEMATICS EDUCATORS

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During this century mathematics educators have striven to develop mathematics curricula that were based on emerging psychological theories of learning and child development. Piaget, Bruner, Gagne, and Ausubel are only a few who have identified models and variables that have led to productive research in mathematics education. One set of psychological variables that is presently being investigated by psycholinguists might further our knowledge of how children learn mathematics. These variables are concerned with how children learn, use and comprehend language, and the relationship of language acquisition to concept formation.

One result of this investigation of child language acquisition has been an understanding of the monumental task that each child masters as he learns his language. Chomsky (1975) states:

> A human language is a system of remarkable complexity. To come to know a human language would be an extraordinary intellectual achievement for a creature not specifically designed to accomplish this task. A normal child acquires this knowledge on relatively slight exposure and without specific training (p. 4).

The child not only masters the syntax of his language, but he does most of it by the age of six. Carol Chomsky (1974) observes that children's "...spontaneous speech at this age (six) rarely provides evidence of discrepancies between their grammar and adult grammar" (p. 60). Language is such an integral part of the human condition that the importance of the child's achievement in mastering it is often taken for granted.

Dale (1976) describes two fundamental insights that underlie recent research on language development.

> The first is the realization that the child does not merely speak a garbled version of the adult language around him, handicapped by limited attention, memory span, and other psychological deficits. Rather, he speaks his own language, with its own characteristic patterns. . .The second insight is that the child himself must act as a linguist: He is faced with a finite set of utterances, many of them ungrammatical. . .from which he must extract the underlying rules in order to be able to use the language creatively for the remainder of his life (p. 2).

The rules of the language--its grammar--are only one aspect of language that the child must acquire. The child must also continually deal with assigning meaning to new words if he is to use language to communicate.

Even though by six years of age the child has learned the grammar of the language, he continues to add words to his vocabulary and to develop meaning for these words. One needs only to scan an elementary school mathematics textbook series to realize the number of mathematical terms that each child is expected to master. Acquiring meaning for mathematical terms is an important part of the mathematics curriculum. In order to design instruction that will assist children in this task, it seems reasonable to examine how the young

child acquires word meaning without instruction.

The two major purposes of this paper are 1) to describe one particular theory of how children acquire meaning for words, namely, the Semantic Hypothesis; and 2) to review some of the research that has tested this theory, specifically in regard to children's acquisition of dimensional and relational terms. These terms provide a focus because they are part of the child's beginning mathematical lexicon. The Semantic Feature Hypothesis is examined as a suggested method for further investigation of children's acquisition of other mathematical terms.

A third purpose which arises from the first two is to emphasize the need for mathematics education research focusing on linguistic variables as well as on cognitive ones.

There is no one generally accepted theory of linguistic meaning, but Katz and Fodor's (1964) theory based on the notion that each word is a collection of basic elements of meaning is the best known and most widely investigated. In this theory the meaning of a word consists of 1) a set of semantic features, each one expressing a part or a component of the meaning of a word, and 2) selection restrictions that limit the possible combinations of words. Some of the semantic features of bachelor, for example, are (Human), (Male), and (Unmarried). The phrase bachelor's wife violates a selection restriction because bachelor includes the semantic feature (Unmarried) and wife includes the semantic feature (Married), and such a phrase would be unacceptable to a native speaker (Dale, 1976).

A semantic field is a set of lexical items (words) that are related in meaning; they share some of the same semantic features. Dimensional terms comprise one semantic field. Children's understanding of dimensional adjectives and relational terms has been the focus of recent research in child language acquisition because this area of study is considered likely to yield information concerning children's semantic structures. It is of interest to mathematics educators in that the words studied are necessary for discussing beginning measurement and quantity concepts. Piaget has provided insight into the child's cognitive development in these areas. It is now time to examine the child's meaning of the pertinent lexical items and to relate this to his cognitive development.

Some of the spatial adjectives that are members of the field of dimensional terms are considered to represent opposite directions on a measurement scale. In the length scale, long is considered to be positive with respect to some accepted standard and short is considered to be negative with respect to the same standard. The negative term is usually considered to be of the form not (X) where X is the related positive term, hence short is equivalent to not (long) Big and wide are positive terms and their opposites, little and narrow, are negative (Clark & Clark, 1977).

In some pairs of dimensional terms, the positive term is considered to be "unmarked" and the negative term is considered to be "marked" (Clark & Clark, 1977). Each dimension is measured from some primary reference and has one or more secondary standards or reference points against which an object is compared. The unmarked term names the measurement along a

positive direction from the primary reference point. The marked term measures from some standard back to the primary reference point or the zero point (E. Clark, 1973). The unmarked term is the one that at times neutralizes to denote the whole scale and not just the positive direction; also it is the term used in "measure phrases". The negative term never neutralizes in either of these ways. For example in the pair long/short, long is the positive term and it neutralizes, it can denote the whole scale of length. There is no corresponding scale called shortness. Also the measure phrase two hours long is used but not the phrase two hours short.

Dimensional adjectives also differ in terms of semantic complexity (H. Clark, 1973). Semantic complexity is a function of 1) the number of restrictions that are placed on the use of an adjective and 2) its number of semantic features. Big/small refer to size in general. There are few constraints on their use for they can be used with one-, two- and three-dimensional objects. Long/short are more complex because they are restricted to a one dimensional extension of length in the non-vertical direction. Clark & Clark (1977) state that tall/short are of equivalent complexity to long/short; the only difference being that tall/short refer to the extension of length in the vertical direction. Previously, H. Clark (1973) described tall/short as being more complex than long/short because tall/short presuppose three-dimensional objects, even though it can be "glossed over as 'vertical length'" (p. 38). Thick/thin, wide/narrow, and deep/shallow are even more complex because they are restricted to secondary dimensions. Depth is a vertical distance downward from ground level. Width is applied only after length or height has been established. Thickness is applied only after length or height, and width have been established (H. Clark, 1973).

Clark and Clark (1977) report the results of research on children's comprehension and production of dimensional terms. They report that the order of their acquisition is:

"a. big-small
b. tall-short, long-short
c. high-low
d. thick-thin
e. wide-narrow, deep-shallow" (p. 499).

The order of acquisition corresponds directly to the relative semantic complexity of the words.

E. Clark (1973) has developed a theory called the Semantic Feature Hypothesis (SFH) which describes how the child acquires meanings for words. When a child begins to learn a word he has only a partial meaning for it in the adult sense. He begins by associating with the word only some of the features that the adult uses to define the meaning of the word. The child's semantic development entails adding more features to his meaning until it corresponds to the adult's meaning. As soon as the child has acquired at least one feature of the meaning for a word he may begin to use the word, but his meaning for the word is simply the features he has acquired. Therefore, only some of his uses of the word will be appropriate. He is not aware that he doesn't have the full meaning of the word.

Psycholinguistic Studies

Some words are closely related in meaning; they have semantic features in common. Antonyms share all features except for the contrasting value of polarity. In order to differentiate between antonyms the child needs to attend to this one difference. Because of the many common features shared by antonyms, at some point in their acquisition the child is likely to consider them as having the same meaning. In reviewing the research that forms the basis of this theory, E. Clark (1973) claims that there is evidence to indicate that the unmarked member is more likely to be acquired first, and that both adjectives in a pair are likely to take on the feature (+Polarity).

Other words in the same semantic field also share a number of features. Some words in some semantic fields can be thought of as organized hierarchically from most general to most specific. The more specific terms will share most, if not all, of the features of the more general terms and will also possess more specific features as well. Therefore, at some point the child will probably treat the specific and general terms as synonyms for the general term until he has acquired the additional specific features that differentiate the meanings of these related words.

The Semantic Feature Hypothesis is based on the following assumptions:

> (a) the first semantic features that the child uses are liable to be derived from the encoding of his percepts, and (b) at a later stage, as the child learns more about the structure of his language as a whole, he will learn which percept-derived features play a particular linguistic role (e.g., animacy) and which are relatively redundant within a set or combination of features. Since the adult also uses perceptually derived information, many of the semantic features used by the adult are also used by the child. The main differences will lie in the fact that the child has to learn the rules of combination for each lexical item.
>
> . . .The growth of lexical entries for word meanings, therefore, makes no claims about what the child actually perceives, only about the sorts of meaning that he will attach to a word in the very early stages of language acquisition (E. Clark, 1973, pp. 74-75).

In summary, the Semantic Feature Hypothesis predicts 1) the order of acquisition of semantic features for individual words (lexical items) and 2) the order of acquisition of words in a particular semantic field. In applying this to the spatial adjectives of the dimensional field, the SFH predicts that the general dimensional feature (Size) will be acquired first, followed by a specific dimensional feature such as (Length), (Width) or (Height), then followed by its polarity feature (+Polarity) or (−Polarity). Also, because marked adjectives are semantically more complex, for any pair of spatial adjectives the positive (i.e. unmarked) term will be acquired first. When the child begins to acquire the negative term, it will be used as a synonym rather than as an antonym of the positive term. Later the child correctly differentiates between the two terms.

Item 2 in the SFH implies that the adult meanings of some of the lexical items in the dimensional field will be acquired in the order of their semantic complexity which has already been stated.

E. Clark (1977) did an extensive review of research from 1968-1973 on children's first language acquisition. The following studies on dimensional and relational terms are from this review. Clark (1972) tested children's knowledge of the antonym pairs that make up the field of dimensional terms by having the child supply the antonym for words given by the experimenter. Children's main error was substituting a semantically simpler term for a more complex one but not vice-versa. Big was substituted for tall, long, high, old, thick, deep, and wide; and small or little was substituted for low, narrow, thin, and young.

Relational terms are of special interest since they have long been used in Piagetian research on the development of conservation. Donaldson and Balfour (1968) investigated 3-year-olds' comprehension of the terms more and less. They found that all of the children in their sample gave correct responses to more but consistently treated less as a synonym for more. Palermo (1973) replicated Donaldson and Balfour's study and carried out a second experiment with 5-, 6-, and 7-year-olds. Nineteen percent of the 7 year-olds still treated less as if it meant more. The percent making this error increased as age decreased. Donaldson and Wales (1970) studied children's comprehension of the terms different and same. Their results are similar to those for more and less in that different was usually interpreted as if it meant same. They also tested children's comprehension on the superlative and comparative forms of a number of dimensional pairs. Children more often responded correctly to the positive, or unmarked, end of the dimension.

The preceding studies are only a few on which E. Clark based her Semantic Feature Hypothesis, others are reviewed in E. Clark (1973).

Since Clark's review of the literature, several significant studies have been completed in these and related areas.

Donaldson and McGarrigle (1974) investigated 3- to 5-year-olds' understanding of the terms all and more. The overall results were that one-third of the children studied used fullness rather than length or number to make decisions about more. Fullness was also involved in children's responses to the all questions. Eilers, Oller, & Ellington (1974) investigated 2- and 3-year-olds' comprehension of the terms big/little, long/short, wide/narrow. The purpose of the study was to investigate two aspects of Clark's Semantic Feature Hypothesis 1) the order of acquisition of spatial terms from general to specific and 2) the marked term being first acquired as a synonym of the unmarked term. The order of predicted difficulty was supported by the data: Big/little were easiest, followed by long/short, and then wide/narrow were the most difficult as evidenced by the number of errors. The prediction associated with markedness was not borne out. There were significantly more errors on the unmarked pair members, and this is directly contrary to the Semantic Feature Hypothesis. Eilers, Oller and Ellington (1974) then conducted another study to see if this result was due to children's natural preference for the smaller item (the item with less extent). The results of the test for size preference was

significant with children more often choosing the objects corresponding to the marked adjectives; i.e. the smaller ones. Therefore many of the children's errors might be attributed to non-semantic preference. When preference was controlled for, the children did equally well on unmarked and marked adjectives. This result is still contrary to the part of the SFH that refers to markedness. Bartlett (1976) investigated 3- to 5-year-olds' comprehension of the terms big/little, long/short, tall/short, and wide/narrow. Her results when using one-dimensional stimuli were very similar to those of Eilers, Oller, and Ellington (1974). Pairs of adjectives were acquired in the order predicted by the Semantic Feature Hypothesis but the main effect for polarity was not significant. Older children made significantly fewer errors than did younger children. Bartlett also tested the children using two-dimensional stimuli. The effect of this was that positive terms were easier overall than negative ones. The order of acquisition for the pairs of adjectives was the same. Bartlett next tested children aged 2-3 on their comprehension of big/little. The purpose was to investigate the SFH prediction on the first pair of spatial adjectives that children acquire. Perhaps after acquiring the first marked adjective, the rest are easier. There was no significant difference, but in looking at the data all of the children who attained perfect scores on big did the same on little. Four children who got a perfect score on little did not get a perfect score on big. This trend in the data is reminiscent of Eilers, Oller, and Ellington's (1974) results.

A series of related studies, though not dealing with dimensional adjectives, was conducted by Kuczaj & Lederberg (1977). They investigated 3- to 6-year-olds' comprehension of the terms older, younger, and bigger. They were mainly interested in whether older meant taller and younger meant shorter, and whether childrens first meaning of younger would be older in agreement with Clark's SFH. The unmarked adjective older was not generally acquired first and the meaning of the marked adjective, younger, did not consistently take on the meaning of older. The meaning of older and younger were related more to size than to age.

Townsend (1976) also looked at children's comprehension of marked and unmarked adjectives. He tested both the comparative and superlative forms of pairs of adjectives where one was marked and one was unmarked (Shorter/taller, lower/higher, thinner/thicker, and less-fewer/more) and of two pairs that were simply opposites (lighter/darker, Skinnier/fatter). The results were that children had more difficulty with some marked adjectives and with negative adjectives that are not marked than they did with unmarked adjectives and the positive adjectives. But their errors on marked and negative adjectives were generally random. Again, the SFH was not supported by the data.

Clark's hypothesis concerning marked adjectives being first acquired as synonyms of their related unmarked adjectives was not supported by most of the studies reviewed. What has been shown is that some children do go through a stage of using some antonyms as synonyms. The meaning of the adjectives when used as synonyms seems to be more universal for some pairs

(more/less), and more varied for others (younger/older). How can the differences be explained? Age might be a factor, as well as natural preference, and the individuality of each child's conceptual and linguistic knowledge. Kuczaj & Lederberg (1977) address themselves to this issue:

> How then can the data be accounted for? A theory based on the child's active interpretation of his linguistic AND non-linguistic environment seems the most viable candidate. . .Basically, the theory suggests that a core part of the language acquisition process is the child's active employment of his cognitive skills and strategies in his attempt to interpret linguistic information, the interpretation being influenced by non-linguistic contextual information as well as the child's current linguistic knowledge and conceptual skills. The acquisition sequence of the meaning of related words (such as antonyms) will vary, then, according to both the child's experiences with the words and the type of relation which holds between the words. The meanings of some words are such that different experiences (or different interpretations of the same experiences) with the words will result in different acquisition patterns (Kuczaj, 1975; Nelson & Bonvilliam, 1973), while the meanings of other words are related such that different experiences fail to result in different acqauisition patterns, thus resulting in surprising consistency from child to child (Haviland & Clark, 1974; Kuczaj & Maratsos, 1975) (pp. 403-4).

The studies did support the hypothesis that acquisition of words in the dimensional field would be from general to specific. Also, specific terms were sometimes treated as being synonymous with more general terms.

The Semantic Feature Hypothesis provides direction when analyzing word meanings into subunits of meaning (features). The studies testing the SFH suggest protocols for assessing the child's possession of appropriate features. The theory, the resulting analysis of word meaning into semantic features, and appropriate testing seem to lead to a better understanding of children's acquisition of the words in a semantic field. It seems that this would be a valuable approach to use when investigating other semantic fields. In the near future I plan to use a similar analysis of word meaning to investigate young children's meaning of some of the terms in the semantic field composed of fractional terms.

Pre-schoolers comprised most of the subjects tested in the reported studies. E. Clark (1973) discusses the Semantic Feature Hypothesis in terms of young children's semantic development. Does a similar kind of development take place when elementary school children are introduced to the words in an unfamiliar semantic field, for example geometry and measurement?

The terms investigated in these studies form a part of the "language of mathematics" that is also a part of the natural language of the child. The beginning concepts that are related to these terms have been extensively studied by Piaget and his followers. It is generally accepted that the child's cognitive structures change with age, hence his concepts also change. The research discussed in this paper indicates that the child's meaning for words

also develops over time. This leads to the conclusion that both concept development and the meaning for the words that serve as labels for these concepts need to be investigated.

Consider the following child's responses to the standard conservation of length task where two identical sticks of equal length are compared and then one is moved forward:

Ruf (4;6). Before staggering: "They're the same length.--
(One stick is moved.)--It's bigger because you pushed it.
The stick is longer. (Piaget, Inhelder, & Szeminska, 1964).

What are Ruf's meanings of the words length, bigger, and longer in these responses to the repeated question: "Are they the same length or is one longer than the other?" (Piaget, Inhelder, & Szeminska, 1964) There is no way to tell from the information given. Can it be assumed that Ruf is using the full adult meaning of each of these terms? The preceding review of the literature does not lead to that conclusion although it is possible. What are other possibilities? Ruf might be associating length, bigger, and longer only with the feature (Size), or he might be using bigger and longer as synonyms, not yet possessing the specific feature that differentiates the two. Ehri (1976) reports that some children do not distinguish between big and bigger. Could Ruf be using bigger and longer as synonyms for big and long? The point is that it is difficult to determine the child's exact meanings of these terms from the child's statements. This discussion is intended to raise some questions in order to emphasize the need to investigate the child's word meaning when investigating his conceptual development. Use of a word by a child cannot be accepted as indicating that the child possesses a full meaning for the word. Judgments are often formed as to a child's conceptual development based on the words he uses and how he uses them. This is a double-edged sword since a child can use words without possessing either the underlying concept or the full meaning of the word; also the child might not possess a word but still have the related concept in some stage of development in his cognitive structure. More research is needed to discover the relationship between these two areas of the child's knowledge, cognitive and linguistic.

Some research has been aimed at examining this relationship. Ehri (1976) investigated the relationship of seriation to children's comprehension and production of adjectives. One result of the study is that:

> . . .findings provide some evidence indicating a relationship between linguistic-conceptual development and cognitive development. Specifically children who have achieved the stage of intuitive seriation were found to utilize a more extensive adjective lexicon in describing object differences, and their comparative language was more advanced than that of non-seriators. In addition seriators displayed greater ability to decentre, as reflected in their superior comprehension of coordinated language (p. 380).

Ginsburg (1977) describes some children's knowledge, reported by others, of various items in the number lexicon (number words) that differ from adults' meanings. He reports 5-year old Steven associated the word five only with sets

arranged in an X-shaped array. When discussing a row of five ducks Steven said: "It isn't five; there isn't one in the middle." (p. 23) Charles when counting five marbles, four red ones and one blue one, counted the blue one last so that it was called "five". For a time afterwards, Charles called any blue marble five. Moira at 4 years of age also used a number word as a name for an individual object. Catherine believed that because she was 4 years old her number was four. Therefore, in a counting situation only four could be used to designate her. Ginsburg concludes: "The accurate use of words does not guarantee understanding" (p. 29). These results are in accord with the general pattern of acquisition of word meaning by children. Ginsburg's discussion of children's meaning of number words is very interesting, but it does not indicate whether there exists a general pattern by which children acquire the meaning of the number lexicon.

Could the numerical lexicon be investigated in terms of the Semantic Feature Hypothesis? What are the features of the number words five and four? Are there perceptual patterns that are related to features for number words as there are for dimensional terms? Beilin (1975) claims that knowledge of the number lexicon based on semantic features alone is inadequate.

> Developmental progression in lexical knowledge involves not only differentiation of meaning as suggested by E. V. Clark's (1970, 1971) theory, but involves integration of meaning elements into a more encompassing logical system. In addition, at least for those parts of the lexicon embodied in number, the underlying structure of the lexicon is part of a large, logical framework that is cognitive and only secondarily linguistic (p. 142).

The preceding statement was based on the results of an extensive study by Beilin (1975) that investigated the development of the child's number lexicon and its relation to the child's developing cognitive capacities. Investigated were four classes of number words: Cardinal-determinate, ordinal-determinate, cardinal-indeterminate, and ordinal-indeterminate. Knowledge of the number words were assessed for five categories of meaning. This study provides a great deal of information about the child's developing number lexicon.

In his interpretation of the results Beilin (1975) claims:

> The convergence or general synchrony between developments in the acquisition of lexical knowledge and those of cognitive development suggests, although it does not provide incontestable evidence for, the assertion that acquisition of the lexicon is under the progressive control of developing cognitive structures (p. 151).

When considering this conclusion it is important to note that the cognitive development of the children in the sample was not investigated. This conclusion was arrived at by comparing the data collected on the knowledge of the number lexicon for children from ages 2 through 7 years of age to the Piagetian theory of cognitive development and to Piaget's stages of development with their approximate age ranges.

Beilin's conclusion is in accord with Piaget's view (Piaget and Inhelder, 1969; Piaget, 1976) that the child first develops concepts and mental structures and then acquires the language that symbolically represents the developed structures. This is not the only accepted theory of the relationship of thought and language. Vygotsky (1962) was critical of this aspect of Piaget's theory. Vygotsky agrees with Piaget that thought and language stem from different genetic roots in humans, but Vygotsky (1962) states that at a certain point in time language and thought meet ". . .whereupon thought becomes verbal and speech rational" (p. 44). Vygotsky (1962) claims that from this time to adolescence, words become increasingly more important as a means of concept formation. No accepted theory as yet exists that describes the exact relationship between thought and language, but it is obvious that a relationship does exist.

In the mathematics classroom the teacher uses language when he/she teaches new mathematical content and reviews previously taught content. Perhaps manipulative materials and other stimuli are also used, in which case the language might differ. But in both cases the child must bring meaning to the words spoken--to the natural language and to the words that are specifically mathematical terms. The child will understand what is being said in terms of his current meanings for the spoken words and the total situation in which the words are used. The research presented in this paper indicates young children do not have full meanings for many very common dimensional and relational terms. This lack of full meaning for words extends to other semantic fields besides those discussed in this paper. With age and experience children do learn the full meaning of words for which they previously had partial meaning. But they are constantly being exposed to new words. So at any given time there exist some words in their lexicon for which they have only partial meaning. This is true for adults as well as children.

In order to increase the effectiveness of instruction, teachers should be aware of the range of meanings that children at a certain age can have for mathematical terms. The Semantic Feature Hypothesis provides only one model for studying children's acquisition of meaning. Some areas of needed study in relationship to this model are: 1) to identify the set of semantic features for different semantic fields of mathematical terms; 2) to identify the order of acquisition of semantic features for specific mathematical terms; and 3) to describe the number and types of encounters or experiences with mathematical terms that are necessary for acquiring a full set of semantic features. The last item is bound to vary for individual words. Are the needed number and types of exposure to a word a function of the number of semantic features in the meaning of a word, the semantic complexity of the term, and/or the nature of the concept that it labels? In addition, other models of semantic acquisition of mathematics language need to be explored by mathematics educators.

The nature of mathematics, the cognitive structure of the child, and the child's linguistic competence are all factors in the child's learning of mathematics. Currently more is known about the first two factors than the third. Until recently there have been few linguistic theories to guide research

in this area. With the surge of research in the area of child language acquisition and other areas of linguistics, that situation has changed. The language component as it interrelates with the child's cognitive development in the learning of mathematics must be studied in order to add a needed component to our understanding of the whole context in which the child learns and uses mathematics.

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DIAGNOSTIC MATHEMATICS WITH DEAF LEARNERS: SOME ISSUES AND POSSIBLE SOLUTIONS

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Background

Passed by Congress in 1975, Public Law 94-142 represents what appears to have been a landmark decision which strengthens the right and privileges of handicapped children for equal education opportunities. As stated by the Council for Exceptional Children (1976), a primary purpose of this act is to guarantee a free public education which considers and takes into account the unique needs of handicapped children. In short, one of the most crucial intended outcomes of this legislation is to present an educational environment in which each handicapped child has the opportunity to develop to his or her full capability.

In view of this mandate, it would seem that the critical role of educational diagnosis in an academic program could take on even added impetus. Indeed, the time appears ripe for a detailed study of the diagnostic and prescriptive process; research we hope would result in the creation or development of diagnositc instruments which could not only provide gross approximations of cognitive, affective and other indices, but, in addition, yield specific information useful in helping to meet the unique needs of handicapped children.

Statement of Purpose

This paper presents an overview of some research questions thought to be central to the diagnosis of deaf students in mathematics. Two major sections follow. The first section reports a review of related literature, the second section proposes research questions that seem to warrant consideration for empirical test. Both parts are included so that together they may function as one steppingstone toward an increased, organized body of knowledge. From this set of conceptual statements, it is further intended that concentrated effort can result in the eventual development and widespread use of improved diagnostic instruments for use with deaf students in mathematics.

Review of Literature

Introduction

The studies cited below clearly reflect a broad spectrum of orientation. While none of the first four (Brueckner, Sovchik and Heddens, and Speer) deal specifically with mathematicsl diagnosis of the deaf, each, nevertheless, raises questions and presents suggestions which may be of especial importance in the diagnosis of deaf students.

Moores conducted a comparative, longitudinal study of eight preschool programs for the deaf. The data and explanations included in this detailed work could be of assistance in the diagnostic model-building process.

The three studies by Johnson, Graeber, and McMurray, respectively, report bibliographic or survey searches and are thought to constitute a significant portion of this literature review. Only the first, by Johnson, describes mathematics programs and materials currently used with the deaf.

Suppes evaluated studies pertaining to cognition in handicapped children. His review and recommendations are structured around the areas of language, concept formation and elementary mathematics.

Cawley offers ideas for building a secondary mathematics curriculum for the handicapped. He reviewed current, representative practices in diagnosis and, step-by-step, describes his use of concept inventories, interviews, remedial modules and other project components.

Barron proposes a strategy for presenting word problems, involving the use of addition and subtraction, to deaf children. This strategy includes ideas for strengthening skills in reading, computation and mathematical understanding.

The final two studies, by Karchmer and Jensema, are demographic and, in part, describe the relationship between deaf students' performance in math and selected variables, such as age at onset of hearing loss.

Diagnosis in Mathematics

In what has become a classic in the field of diagnosis and prescription, Brueckner (1935) stated that the process of diagnosis in arithmetic should be based upon computational, informational, sociological, and psychological questions. Factors that affect proficiency in arithmetic, according to Brueckner, include those which are physiological (such as hearing), mental, pedagogical, social/environmental, and emotional.

Kratochwill and Severson (1974) found traditional approaches to diagnosis lacking due to heavy emphasis placed on "test based" assessment. Quite in accord with Brueckner's broader interpretation of diagnosis, Kratochwill recommended a process approach which considers knowledge, learning style and rate. He emphasized that such an approach must be dynamic and sufficiently flexible to adapt as the student interacts in different learning environments.

Sovchik and Heddens (1978) pointed to a difficulty raised by several others in this review; that is, the need to pay much more attention to concept formation than to skill development. Toward this end, they suggested that informal classroom diagnosis can be strengthened through the use of assessment technitques in concrete situations. The authors further stated that regarding remediation, children too often fail to see the link between computational examples and meaningful situations which actually occur in life. Remedial activities should be selected on the basis, at least in part, of childrens' abilities to clearly portray the relationship between mathematics and its application in the real world.

In a presentation at the Fourth National Conference on Diagnostic and Prescriptive Mathematics, Speer (1977) began by referring to the woeful lack of substantive research which dealt specifically with diagnosis of mathematical difficulties. He outlined and described a theoretical model, for use in essentially clinical situations, which takes into account: (1) mathematical (content) deficiencies; (2) mathematical style; (3) educational cognitive style.

Speer closed by posing several significant questions which he felt should be addressed concurrent with the actual use of his diagnositc model. One such question has been frequently cited by others on numerous occasions: if a

student fails to perform adequately on a diagnostic instrument, is it because of (other) factors which may affect the clinician (teacher)-student realtionship?

Moores' Evaluation Study

Moores (1974) conducted a comparative, longitudinal study of eight preschool programs for deaf children. Programs were selected from across the country to represent a diverse range of educational methods and ideologies. One of several tests administered was the Metropolitan Achievement Test. On the arithmetic subtest, deaf children across programs performed less well than their hearing counterparts. Moores felt that the verbal nature of the test may be used to explain this performance.

Bibliographic or Survey Searches

Johnson (1977) observed that, due to the scarcity of published mathematics materials for deaf students, school personnel are continually required to develop their own tests and materials. To improve on this situation, Johnson conducted a survey with the hope that its findings could be generalized and serve as a benchmark for research in the areas of mathematics education for deaf students.

About one-half of the lower through upper schools responding to Johnson's survey indicated the use of a specific mathematics curriculum based on one or more of the following: state mandated, locally produced, textbook series, other. Approximately 50% of the classroom period was devoted to the development of skills. In contrast, use of individualized instruction was reported to have been used less than 5% of the time. Regarding the use of materials, a wide range and number of textbooks was used in schools for the deaf. These books were used principally for example problems and drill practice exercises.

Focusing on diagnostic-prescriptive activities for K-8 teachers, Graeber and Baxter's (1967) literature report was designed to provide a knowledge base for classroom intervention or research efforts. Several references cited relate particularly to special education.

McMurray's literature research (1976) dealt specifically with learning disabilities. This report focused on areas of theory, diagnosis and assessment and remedial techniques. Each of these was broken-down according to the following schema: general neurological-perceptual impairment; reading; speech and listening; spelling, mathematics and writing.

Suppes on Cognition in Handicapped Children

In a significant contribution, Suppes (1974) conducted a literature review on cognition and its relationship to the development of academic skills in handicapped children. The review was sectioned into three main parts: language skills, concept formation and abstraction, elementary mathematical skills.

The one difficulty most pronounced for deaf children relates to their proficiency in a standard natural language. Consistent with Jensema's (Note 2) findings, presented later in this article, Suppes stated that the seriousness of this difficulty is a direct function of both the extent of hearing loss and the age at onset of the hearing loss.

Several studies are reported which center around the long-debated

question of oral vs manual method of language training. A critical item from the standpoint of acquisition and functionality is the relationship between some sign system (e.g., signed English, American sign language (ASL)), and English as a natural language. Suppes' recommendation, therefore, was twofold: (1) that investigations be undertaken which study the relationship between these alternative systems (modes) of communication, and (2) that additional research be conducted to explore the parallel between these systems of communications and natural languages. Moores' evaluation study cited earlier is one beginning attempt in this direction.

Regarding the area of concept formation, two of Suppes' observations seemed to stand out: (1) a need to systematically study the role of language in concept formation and (2) a need to test the ability to manipulate complex inferences in nonverbal situations through the use of sign language. He proposed that further research be initiated which would explore the relationship between successful prior experiences in sign language and the effect these experiences have on the use of a written natural language.

In the final section, which dealt with arithmetic skills of deaf children, Suppes reported data collected from the drill-and-practice programs conducted through the use of Computer Assisted Instruction (CAI) at the Stanford Institute. Two conclusions flow from this experiment. First, variables or factors which affected performance of hearing students (regrouping vs. no regrouping) affected deaf children in a similar manner. Second, cognitive performance of deaf children seemed to be significantly enhanced when associated learning tasks did not principally involve the use of verbal skills.

Cawley's Secondary Curriculum

Cawley (Note 1) developed a curriculum proposal in mathematics for children with learning disabilities in the sixth through ninth grades. Across four content strands, Cawley's proposed curriculum included: (1) screening and diagnostic procedures, (2) short-term and long-term instructional components, (3) individual and group record keeping systems, (4) two and three dimensional instructional materials, and (5) a teacher guide.

Included in his review of the literature was an examination of a number of commercial diagnostic tests, one of which is titled Key Math (Connolly, 1971). Cawley described the inadequacy of this test to be its failure to provide specific indicators as to where and how remediation should occur.

Cawley was also critical of the failure of diagnostics in general to consider the learner's most and least efficient modes of performance. He felt it is ironic that the use of concrete, iconic, and symbolic embodiments pervaded throughout so many instructional mathematics programs, yet the design of accompanying diagnostics was not sufficiently flexible to uniformly capitalize on a given learner's performance style.

Addition and Subtraction Word Problems

Borron (1975) analyzed verbal problems in addition and subtraction and suggested procedures for helping deaf children learn to solve subtraction problems. Verbal problems can be introduced informally through the use of questions about actual classroom situations. The researcher suggested that students write or tell stories about mathematical sentences. Formal

presentation, according to Borron, should proceed from concrete to symbolic through the sequential use of dramatization, pictures, set drawing, and visualization.

Demographic Studies

In a longitudinal study of approximately 1500 deaf students, Karchmer (Note 3) found that students averaged slightly over two grade equivalents growth in mathematics computation during a three year period as indicated on the Stanford Achievement Test (SAT). While level of mathematics achievement increased by age, there were consistent indications that younger groups gained faster than older groups. Karchmer explains that this may be due to the fact that the lower levels of the mathematics subtest have less verbal content than the upper levels.

Jensema (Note 2) examined the relationship between academic achievement and selected demographic characteristics of deaf children. In his examination, Jensema used the results of the 1973 Stanford Achievement Test, Special Edition for Hearing Impaired Students (SAT-HI), administered to a sample of almost 7,000 students enrolled in special education programs for the deaf.

For mathematics concepts, the average scaled score for all age groups was 128.2, which corresponds to a medium raw score obtained by a hearing student at the 2.8 grade level. For mathematics computations, the picture appeared brighter. The average scaled score for all age groups was 145.3, which corresponds to a median raw score obtained by a hearing student at the 4.5 grade level.

As indicated in a parallel fashion by Karchmer, the demographic variables 'age at onset of the hearing loss' and 'degree of hearing loss' seemed to have less of a relationship in mathematics computation than they did in other areas more dependent on language skills. One interesting result occurred in conjunction with the variables 'additional handicapping conditions.' In this instance, scores of students reported as having additional handicapping conditions tended to be lower for the mathematics subtests than for the language tests. Jensema concluded that factors in addition to language deprivation may need consideration in situations with additional handicaps.

Summary

It is evident that the number of published reports dealing with diagnosis in mathematics is strikingly scant, but corresponding material in deaf education is even less. Still, a number of positive chords persist in this regard and appear equally evident.

First, the trend appears to be on a steady upswing. More and more emphasis is being placed on diagnosis and prescription, both in theory and in practice. From these theoretical and practical gains, an organized body of knowledge relating to mathematical diagnosis may be emerging.

What is encumbent for mathematics educators of the deaf is that a corollary set of sta tements, a body of knowledge, be built. Existing literature on cognition, concept formation, and language should provide some benchmarks. New developments in other fields (e.g., audiology) can have significant impact. Obviously, the strict scope or dimensions of such developments cannot

be measured on an a priori basis; rather, the creativity of those individuals making contributions will determine to a degree the extent to which applicability will occur.

The author feels that the inductive role of empirical studies in shaping the nature of that systematized body of knowledge is difficult to overjudge and consequently should be viewed as a priority. In view of the wide, though limited in number, range of reports presently available, the careful design and implementation of empirical studies might bridge present information and, at the same time, assist in enlarging the mentioned conceptual framework.

Toward the Development of Diagnostic Research Studies

Syntax, Semantics, Pragmatics

According to praxiologists, investigations into the use of a symbol system typically involve scrutiny of at least syntax, semantics, and pragmatics. Syntactic relations, quite simply, are usually thought of as relations between symbols. Relations between symbols and their referents are commonly labeled or described as semantic. Finally, relations between symbols and their intended users are pragmatic relations.

Among a host of research studies which easily could be generated using this classification, two important questions are: (1) under what conditions is the deaf student able to demonstrate knowledge of the sign, yet unable to demonstrate knowledge of the corresponding written word?; (2) under what conditions is the deaf student able to demonstrate knowledge of both sign and word, yet still unable to grasp the meaning or referent involved?

The relation between symbol and intended user may, of course, differ among individuals. Additional research could investigate the relation between a given structure and two or more intended users. This investigation would then be a study of second-order relations, that is, relation between relations.

Additional Handicaps

To maximize opportunities for student response, it is critical that further work focus on the relationship between deafness and other handicaps which may be present. As an example, if a deaf student with an additional handicap of visual impairment is given twenty pictorial problems and fails to perform adequately on the task, are we to assume that such failure can be attributed to a lack of conceptual understanding or perhaps that the manner in which the exercises were presented precluded, prima facie, successful completion?

Perceptual Variability and the Concept of Noise

Another interesting area of research that deserves some attention involves one issue that may affect the ability of children to abstract. This issue is referred to by mathematics educators as "noise" and is often described as the irrelevant characteristics or features of a concept. This concept is regularly discussed in conjuction with the perceptual variability learning principle which states that to promote generalization the conceptual structure of a concept should be portrayed in a variety of different yet perceptually equivalent forms.

Given this orientation, the question may then be posed: to what extent is the diagnostic process with deaf children in mathematics interfered with, or

otherwise affected, by tendencies to concentrate on irrelevant features? It would seem that the presence of additional conditions, such as those which might relate to visual acuity, would make this question all the more germane.

A Final Suggestion

An appropriate culminating suggestion, as it turns out, does not lend itself directly to empirical test; rather it expresses an important need for researchers to adopt a more global view of diagnosis based on a defined set of guiding first principles. In this regard, it seems reasonable that such a position reflect not only the individual character of variables which may come to bear on the diagnosis of deaf students in mathematics, but also the relationship between variables and the extent to which these relationships may affect the diagnostic process.

Conclusions and Recommendations

The purpose of this paper has been to introduce several issues critical to the diagnosis of deaf students in mathematics. The strategies followed included a review of related literature and the description of research problems from which hypotheses could be set forth and investigations undertaken.

Very little literature relating expressly to the purpose as stated was discovered. Instead, a collection of pieces representing a wide state-of-orientation was included. Three broad research problem areas were briefly described. It is important to point out in this regard that those areas cited, admittedly, constitute only a very small portion of territory that needs to be covered. The nature of this presentation, however, precluded introducing numerous other problem areas at least equal in significance or in going into more depth on the three areas discussed.

Probably the single most important recommendation forthcoming from this paper is the request that researchers in deaf education adopt a more comprehensive outlook on diagnosis in mathematics. We should expect, as a result, that given a more inclusive interpretation, such a diagnosis would not only reveal skill and concept proficiencies in mathematics, but also detail other proficiencies of which we have reasonable assurances to believe have to do with performance in mathematics.

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ANXIETY AND MATHEMATICS LEARNING: A SELECTED REVIEW OF THE RESEARCH AND SUGGESTED IMPLICATIONS

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The topic of anxiety has been widely researched in the psychological and medical literature. Speilberger (1972) estimates that between 1950 and 1970 over 5,000 articles or books on anxiety were published. A wide range of theoretical definitions has yielded a lack of established correlations among different definitions of anxiety thus reducing the practical value of the construct. Researchers have investigated chronic or general anxiety as exhibited by the hypertense person. Situation specific anxieties such as test anxiety, fear of high places, and mathematical anxiety have also been investigated (Dreger, & Aiken, 1957; Hill, & Sarason, 1966; Holmes, 1936; Sarason, 1960).

Due to the difficulty of defining a construct such as anxiety, some researchers have advocated studying specific anxieties and corresponding therapeutic treatments (Sarason, 1960). Thus, the construct of mathematical anxiety has emerged as a situation specific form of anxiety.

Research on mathematics learning and anxiety included in this review is classified into three categories: 1.) those studies which developed or suggested possible instruments for detecting mathematical anxiety; 2.) those studies which investigated related variables such as mathematical achievement, sex, or self concept; and 3.) those studies which suggested possible therapeutic programs.

Dreger and Aiken (1957) used a modified version of the Taylor Manifest Anxiety Scale and reported that 35% of the 704 college students sampled exhibited mathematical anxiety. Hill and Sarason (1966) used the Test Anxiety Scale for Children (TASC) in a longitudinal study of anxiety in elementary school children. Hill and Sarason reported that high anxiety was inversely related to arithmetic comprehension and arithmetic problem solving scores. More recently, Richardson and Suinn (1972) have developed a 98 item inventory of situations that might arouse mathematical anxiety. The Mathematics Anxiety Rating Scale (MARS) developed by Richardson and Suinn has yielded test-retest reliabilities of .78. Aiken (1976) suggested that the construct validity of this instrument was promising. Finally, self report inventories, especially at the elementary school level, have been suspect in terms of reliability (Aiken, 1970; Szeleta, 1973).

It has been demonstrated in the psychological literature that complex functioning is affected by high levels of anxiety (Castaneda, Palermo, & McCandless, 1956; Hill and Sarason, 1966; Sarason, 1960; Speilberger, Anton, & Bedell, 1976). The psychological literature also suggests that sex differences in reported anxiety are minimal in the early elementary years but that girls tend to report more anxiety in later school years perhaps because boys are culturally conditioned to defend against admittance of anxiety (Hill and Sarason, 1966; Phillips, Martin, & Meyers, 1972).

Anxiety

In mathematics education investigations, the relationship between mathematics performance and anxiety has been investigated. Szetela (1973) studied the relationship between anxiety and mathematics performance in a junior high school mathematics class, finding that the inverse relationship between test anxiety and mathematics performance was "barely significant". Jonsson (1966) found that for 358 sixth graders, levels of test anxiety significantly interacted with test version to the detriment of the performance of the high anxious students taking the more difficult version. Hellman (1976) reported that for 138 fourth graders and 150 eighth graders reported anxiety was not related to mathematical performance.

Few studies have been reported which prescribe treatments for symptoms of mathematical anxiety. Natkin (1967) significantly reduced anxiety towards mathematics in male and female undergraduates by using a form of desensitization therapy. Programmed instruction has been investigated as a means of increasing achievement of highly anxious individuals. Tobias (1973) has reviewed the research on the interaction between anxiety and achievement for programmed instruction and suggested that the presumed superiority of programmed materials for anxious subjects was largely unsupported. Donady and Tobias (1977) described the development of a clinic for treating mathematical anxiety at Wesleyan University. Donady and Tobias (1977) also commented on societal influences which create a "disproportionate number of math avoiding and/or math anxious females". Fennema and Sherman (1977a, 1977b) concluded that the expectation that males are invariably superior in mathematics achievement and spatial visualization was not supported. The Fennema and Sherman affective scales developed for the aforementioned investigations may be useful in clinical investigations of mathematical anxiety.

In closing, the psychological literature relating anxiety to learning is much greater in quantity than the available literature relating mathematics learning to mathematical anxiety. There seems to be a need for further clarification of the relationship between mathematics learning and mathematical anxiety. Some possible questions are the following:

1. Can reliable and valid instruments be developed so that mathematically anxious students may be identified and treated in the early elementary school years?
2. What are the longitudinal effects of mathematical anxiety?
3. What relationship exists between the mathematical anxiety of teachers and students?
4. What relationship exists between various instructional sequences and mathematical anxiety?
5. What relationship exists between teacher methodology and mathematical anxiety of students?
6. How does mode of instruction (concrete, semi-concrete, and abstract) interact with mathematical anxiety?
7. What population of subjects needs priority therapeutic interventions?

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MODALITY: IMPORTANCE TO DIAGNOSIS AND PRESCRIPTION

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At the last conference it was succinctly pointed out that there is an incredible communications gap between mathematics educators and special educators when it comes to dealing with mathematics learning difficulties of children. (Engelhardt, 1977).

Historically, this gap is the result of the divergent attempts of both bodies to identify the causes of arithmetic learning disabilities in children, and the suggested appropriate diagnosis and prescriptive strategies accompanying each theoretical model.

Mathematics educators traditionally lean toward a task analysis model while those involved in special education are more usually advocates of perceptually oriented models.

A task analysis model is content or discipline oriented. It attempts to assess a child's mathematics strengths and weaknesses by utilizing a content hierarchy, emphasizing component skills and integrating them into more complex terminal tasks. Instruction, then, is geared to strengthening the academic weaknesses indicated in the child's hierarchy.

With perceptually oriented models, the prime concern of initial assessment is to identify ability (process) strengths and weaknesses. Generally, learning modality (auditory, visual, kinesthetic, etc.) dominance or weakness is assessed in order to prescribe appropriate remediation. The primary concern is the identification of perceptive and/or psycholinguistic abilities or processes which are presumed to cause inadequate skill development (Ysseldyke, 1974).

Perceptual processes are those operations of the brain that involve organizing and interpreting stimuli: size, color, shape, texture, sequencing, etc. Proponents of this approach use a variety of tests to determine the brain's ability to process functioning. These tests attempt to measure visual perception and visual-motor integration modality; auditory-perceptual modality; and other tactile, kinesthetic and motoric modalities.

Visual perceptual/motor integration involves spatial relationship, visual discrimination, figure ground, visual closure, visual memory (sequencing), and motoric modality. The most widely used tests in this area are the Marianne Frostig Developmental Test of Visual Perception that contains five subtests: 1) Eye-hand Coordination, 2) Figure Ground, 3) Form Constancy, 4) Position in Space, and 5) Spatial Relations. The Purdue Perceptual Survey Rating Scale includes test areas in 1) balance and posture, 2) body image and differentiation, 3) perceptual motor match, 4) ocular control, and 5) form perception.

Another test is the Bender Visual-Motor Gestalt Test. Older than most, it was originally developed and designed for clinical use with organic pathology. It consists of nine designs which the child copies. In addition to utilization as a visual perceptual instrument, it is also used to diagnose emotional disturbances and to predict school achievement.

Many of the tests in this area also have an available training program that is designed to remediate any deficiencies that the test might identify.

Modality

Auditory perceptual modality includes auditory discrimination, auditory sequencing memory, auditory figure ground, sound localization, and sound awareness.

Few commercial tests are available in the area. The most notable is The Wepman Auditory Discrimination Test that uses the procedure of discriminating like and unlike words in the auditory mode.

Another test used and designed to comprehensively measure language functioning is the Illinois Test of Psycholinguistic Abilities. This test contains twelve subtests which include both auditory and visual categories. It is widely used by special education specialists in diagnosis of possible learning difficulties.

After modal identification, two options are available to the classroom teacher or special education specialist. The first is to provide specific training with a closely related training program that remediates any weaknesses that the text might uncover.

The Frostig Horne Program in Visual Perception is one of the more notable programs. It is designed to remediate the deficiencies uncovered by the Frostig test. Other remedial programs are designed to attend to auditory perceptual problems.

In any case, before the student receives any developmental instruction in the specific cognitive area his modality weakness is attended to.

Hammill and Bartel (1975) state, "Thousands of elementary, special education and pre-school teachers have their readiness, preventive and corrective activities based exclusively or in part on such programs (perceptual-motor)."

The second teaching option advocated by the special educationists is to organize instructional materials and teaching strategies along the dominant learning modality of the child. "Thus instructional efforts would be more fruitful if we taught using the child's dominant learning channel" (Mann-Suiter, 1974).

According to Mann and Suiter (1974), "The teacher must identify the 'open channel,' that is, the modality (auditory, visual, or tactual) which enables the child to learn best. By utilizing the open channels, the teacher can relate instruction associatively. For example, a child with a visual channel problem may learn best through an auditory-to-visual-plus tactual association rather than by a visual-to-auditory association."

Some mathematics educators are also suggesting that modality considerations be an integral part of both diagnosis and remediation.

Doris Homan (1970) writes, "Since visual motor problems cause many learning disabilities in arithmetic, tests for these problems should be administered. Effective tests of this type include the Frostig Developmental Test of Visual Perception, Bender Visual Motor Gestalt Test for Children...."

John Wilson, Director of the Arithmetic Clinic, Maryland University, (1976) states, "As reported at last years' conference, they (S. Florida U.) are using the matrix and a variation of the content taxonomy to codify diagnostic items in a way which reveals a child's modality strengths as well as content strengths. This is a most exciting and potentially powerful application, and we look forward to further reports on their efforts."

Further he states, "In the same vein some of our students at Maryland who are dual majors in mathematics education and special education are beginning to use the matrix as a guide for creating new or modifying existing exemplars so that their sensory attributes are more effective with children with specific sensory disabilities."

Kurtz and Spiker, in the December 1976 Arithmetic Teacher, using both the Illinois Tests of Psycholinguistic Abilities and the Wechsler Intelligence Scale for Children, state:

> "A learning disabled child may be expected to overcome his disability...only if he can be identified at an early age so that a program to fit his specific needs can be formulated and carried out. This, referred to as 'prescriptive teaching,' will scrutinize his disabilities and work on those underlying processes giving the trouble, e.g. auditory, visual, perceptual-motor, and so on."

Martha Hopkins in the April 1978 Mathematics Teacher, "The Diagnosis of Learning Styles in Arithmetic" states, "Unless a teacher is aware of the child's preferred learning style (tactile, auditory, and so on) appropriate experiences resulting in the most efficient learning happen only by accident."

Further, many of the measurement instruments designed to assess mathematics ability relative to the identification and assessment of special needs of children, as mandated by Public Law 94-142, have a strong perceptual orientation. One, for example, is the Massachusetts screening instrument. In its directions to teachers (Southwick, 1976) as developed by the Massachusetts Teachers Association, it advises: "Educational assessments provide the basis on which a student's learning needs and learning style are determined, which in turn provide the basis for designing an educational plan responsive to those learning needs in terms of the student's learning style. Learning style refers to the modes by which the child learns most effectively, i.e. through listening (auditory mode), through seeing (visual mode), through touching (tactile mode), through doing (kinesthetic mode)."

In essence then, many professionals stress the importance of modality as a component for instruction in the diagnostic/remediation cycle. Some insist that before arithmetic instruction begins for the 'learning disabled' child the child should undergo process training. Others stress the need to identify the learning style, or dominant modality, and teach according to that strength.

Certainly, if modality is that important in the teaching/learning continuum, then to ignore it in designing instructional strategies for children borders on the criminal. On the other hand, if it is of no importance then to label children as 'disabled' and waste precious student time with it is equally repugnant.

Two basic issues must be addressed when considering modality as a component in the diagnostic/remediation cycle. Initially, are the measurement instruments used to assess strengths and weaknesses reliable and valid? The second issue is that, assuming modality preference could be isolated, would instruction according to indicated strengths be more productive?

What does research indicate in respect to the first question on those tests previously mentioned: Frostig, Wepman, Bender-Gestalt, and ITPA?

The Frostig Developmental Test of Visual Perception is designed to measure some perceptual quotient that indicates a child's level of visual-perceptive development. A cursory look at cumulative research on this test generally does not support this contention. Most results indicate that this test rapidly decreases in usefulness and achievement prediction after the first grade (Ashlock, 1963; Black, 1973; Colaruso, Martin and Hartung 1975; Liebert and Sherk, 1970; Nielsen and Ringe, 1969; Olson, 1958; Robinson and Schwartz, 1973; Rosen, 1966). Further, some researchers have concluded that in fact the test measures intelligence and not perceptual variations (Corah and Powell, 1963; Gamsky and Lloyd, 1971).

Ross (1977) summarizes much of the research when he states:

> "Unfortunately neither test nor program has stood under the scrutiny of carefully controlled research. The test does not seem to test what it is supposed to test; the program does not seem to result in improved perceptual performance; and neither test nor program seems to bear much relationship to the academic learning problems of children....Uneffected by this lack of sound foundation, test and program continue to enjoy great popularity among educators." p. 26

The Wepman Auditory Discrimination Test is constructed to determine the child's ability to discriminate between sounds. Research is generally mixed with regard to the effectiveness of this test or to what it purports to do (Hafner, Weaver, and Powell 1970; Flynn and Byrne, 1970; Larsen, Rogers and Sowell, 1976; Goetzinger, 1960).

It is interesting to note that a review of literature by Hammill and Larsen indicate evidence which suggests that some auditory-perceptual abilities have little to do with the skill of reading. They (1974) reviewed almost 300 distinct correlation coefficients reflecting the relationship between tests of auditory perception and reading. The median coefficient was not statistically significant at the 5⅔ level of confidence.

The Illinois Test of Psycholinguistic Abilities attempts to measure language development according to input, processing and output abilities. The research involving this test is extensive. Much of this research has been adequately reviewed by Hammill and Larsen (1974). They found some support for this test with the exception of the subtest on visual sequential memory.

Newcomer and Hammill (1975) examined the research on the relationship between academic achievement and ITPA. In essence, "none of the twelve ITPA subtests lacked predictive validity for any aspect of academic achievement studies" (p. 735).

In addition, Newcomer and Hammill reviewed the diagnostic validity of the test by determining how the test distinguished between good and poor readers. When mental ability was controlled, neither the total score nor subtest scores reached the 50 per cent level for successful discrimination of good or poor readers.

The Bender Visual-Motor Gestalt Test attempts to assess neurological impairment. As the oldest test of the 'learning disabled' battery much research is available as to its ability to assess cognitive development. This research seriously questions its ability to successfully meet its objectives (Ackerman, Peters and Dykman, 1971; Hartlage and Lucas, 1976; Lackmann, 1960; Nielson and Ringe, 1969; Obrzut, Taylor and Thweatt, 1972; Robinson and Schwartz, 1973; Wikler, Dixon and Parker, 1970).

Overall, Ysseldyke and Salvia (1974) state that "few measurement devices demonstrate the necessary reliability to be used in decisions regarding differential educational programming." Further, "Lack of reliability raises specific problems regarding identification of ability strengths and weaknesses."

The second issue is that is modality strengths could be identified then most effective teaching would utilize these modality strengths. Much of the research in differentiating instruction according to modality preference is to be found in the area of reading. Studies differentiating reading instruction according to modality strengths (Bateman 1968; Bruininks 1968; Cripe and Wilson 1966; Neville and Vandever 1971; Freer 1971; Smith 1971; Robinson 1972; Waugh 1973) do not support the hypothesis that teaching to modality perference is more efficient and productive.

Larrivee (1975) reviewing accumulative research pertinent to the issue of modality preference in reading concludes:

> "Differentiating instruction according to modality preference does not on the whole facilitate learning to read....adapting the instructional mode to the learning's preferred modality is not important for teaching beginning reading..."

What then are the implications for the teaching of mathematics? Certainly the current trent toward modality identification and instruction is questionable at best.

Until more supportive evidence is available in the area of modality-preference, I suggest mathematics educators follow the advice of Ross (1976).

> "If a child fails to learn, look for a different way of teaching. Don't look for something that is wrong inside the child."

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THE ROLE OF THE TEACHER IN A DIAGNOSTIC-PRESCRIPTIVE INSTRUCTIONAL PROGRAM: IMPLICATIONS FOR TEACHER PREPARATION

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Diagnositc-Prescriptive strategies in teaching mathematics have received renewed interest over the last decade. Today's concern for diagnostic techniques in mathematics is not necessarily a new trend, rather a revitalization of the pioneer work of Brownell, Buswell and John, Weaver, Fernald, Brueckner, Glennon, and others. Contemporary diagnostic concerns include the use of both norm- and criteria-referenced testing, learning hierarchies, a variety of record keeping procedures, modes of representing concepts, measures of affective growth, instructional objectives, and the identification of stages of cognitive development, to name a few. The influence of Popham, Gagne, Bruner, Bloom, Piaget, Glennon, J. Wilson, Ashlock, Heddens, and others has contributed greatly to the careful study of children and their individual progress in learning mathematics. Attention has been given to learning styles, computational errors, defective algorithms, sequencing instruction, testing, mastery and retention levels, and other areas of concern within a diagnostic-prescriptive or clinical program.

In addition to the efforts of mathematics educators, another impetus for the current interest in diagnostic-prescriptive techniques is pragmatism. Countless committees have carefully reviewed declining standardized math scores and National Assessment of Educational Progress results and publicly denounced the instructional programs responsible for these generally low profiles. Today's society has assisted in shaping many students who are generally not highly motivated toward success in mathematics. It is obviously very difficult to carefully sift out all the variables responsible for the mathematics student of today, but it is safe to assume that some of the interest in diagnostic-prescriptive strategies in mathematics is related to the need for remedial and compensatory programs. The National Council of Supervisors of Mathematics note that "The narrow definition of basic skills, which equates mathematical competence with computational ability, has evolved as a result of several factors, of which one is an increased awareness of the need to provide remedial and compensatory programs" (National Council of Supervisors of Mathematics, 1978). The President's interest in the back-to-basics movement is evidenced by his recent proposal (Feb. 1978) to increase ESEA Title I funding in 1979 by fifteen percent. Many Title I programs for disadvantaged children have involved a remedial mathematics component for years. Some of the finest examples of diagnostic-prescriptive mathematics instruction occur in these remedial programs. Other school district programs which may be funded by locak, state, or federal funds provide for both remedial and enrichment mathematics students. Many schools operate their own mathematics lab or clinic emphasizing diagnostic-prescriptive strategies. School district programs, regardless of their level of funding, which emphasize diagnostic-prescriptive techniques are another impetus behind the move for the use of diagnostic-prescriptive techniques in the regular classroom. Most school-district programs

which emphasize diagnostic-prescriptive strategies are staffed by teacher(s) with specialized training and/or additional assistance provided by teacher aides and/or volunteers.

At this point, it is important to recognize the thrust of special educators in using diagnositc-prescriptive instruction within many subject areas at virtually all levels of learning. Many special educators have used diagnostic-prescriptive strategies in their classrooms for years, and the legislation (P.L. 94-142) currently linked with mainstreaming and requiring individualized educational programs is, in fact, emphasizing the diagnostic-prescriptive or clinical model of instruction.

As we continue to examine the current interest among mathematics educators in diagnosis, remediation, enrichment, prescription writing, clinical settings, and clinical research and relate these to the social and educational concerns for accountability, basic skills, mainstreaming, and minimum competence, among others, it is safe to assume that the current interest will be at least maintained at a very high level in the near future.

The interest in diagnositc-prescriptive techniques of mathematics instruction is not confined to school-based instruction. Teacher education in diagnostic-prescriptive strategies is vital to the success of future programs and may provide for a more realistic mathematics methods orientation. It is probable that most undergraduate teacher preparation programs in elementary education consist of one mathematics methods course or component. Unfortunately, many of these methods courses continue to focus on the development of teaching strategies for whole class instruction.

Specifically, the purpose of this study was to collect formative data to show the impact of two teacher-training experiences on low-achieving sixth-grade pupils. It was not intended as an experimental comparison to decide which training procedure was better.

The first experience was designed to train prospective elementary mathematics teachers to perform the professional tasks needed to implement the steps of the Goal-Referenced Instructional model (Diagnostic-Prescriptive or Clinical) shown in Figure 1 for just one of the types of individualized instruction shown in Figure 2, Individually Prescribed Instruction. (Individually Prescribed Instruction does not refer to the IPI Project of the LRDC at the University of Pittsburgh. It refers to the type of instruction described by Trueblood 1971). The second experience trained prospective elementary teachers to implement the procedures set forth in the teacher's guide for the classroom text assigned by a local school district.

TES population. Ten elementary teacher education students (hereinafter referred to as TES) who enrolled in an undergraduate class, Methods for Teaching Elementary School Mathematics, were randomly assigned to teach one of two classes of sixth-grade pupils. These TES had had a CUPM Level I mathematics course but no previous methods instruction.

Pupil population. Because of School district policy concerning pupil assignment to classes, the sixth-grade pupils could not be randomly assigned for instruction. This obviously limits the generalizability of the results, and requires that some care should be taken when interpreting the data.

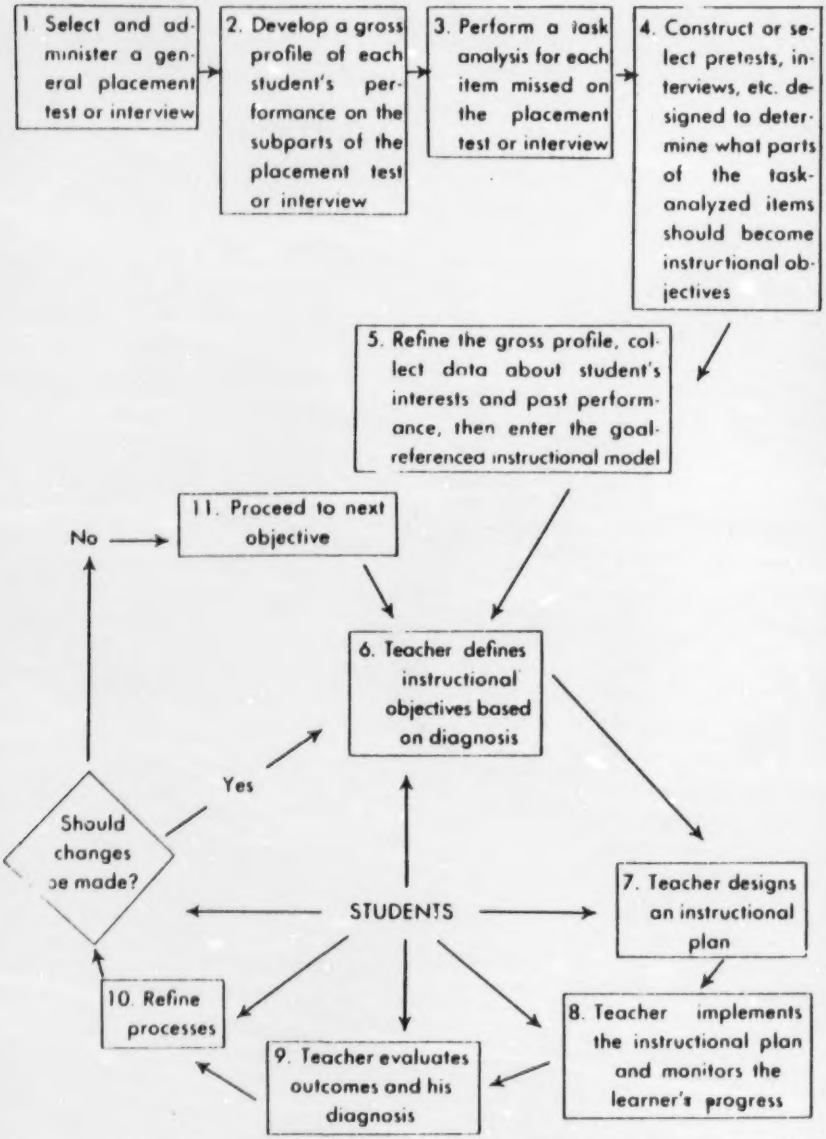
1. Select and administer a general placement test or interview
2. Develop a gross profile of each student's performance on the subparts of the placement test or interview
3. Perform a task analysis for each item missed on the placement test or interview
4. Construct or select pretests, interviews, etc. designed to determine what parts of the task-analyzed items should become instructional objectives
5. Refine the gross profile, collect data about student's interests and past performance, then enter the goal-referenced instructional model
11. Proceed to next objective
No
6. Teacher defines instructional objectives based on diagnosis
Yes
Should changes be made?
STUDENTS
7. Teacher designs an instructional plan
10. Refine processes
8. Teacher implements the instructional plan and monitors the learner's progress
9. Teacher evaluates outcomes and his diagnosis

Figure 1. A goal-referenced instructional model (Diagnostic-Prescriptive or Clinical) to individualize instruction in the elementary school classroom.

Source of instructional means	Source of instructional objectives	
	Teacher	Learner
Teacher	Individually prescribed instruction	Personalized instruction
Learner	Self-directed instruction	Independent study

Figure 2. A matrix showing the source from which objectives and instructional means originate for four types of individualized instruction.

Mathematics scores on Form W (May, 1971, adminstration) of the Stanford Achievement Test, together with their fifth-grade teacher's recommendations, identified the pupils as low achievers in mathematics. The school administrator and teachers had placed these pupils in the three classes through a random selection process. Of the 47 pupils in the study, 60% scored below the fifth stanine on all subtests of the Stanford Mathematics Achievement Test. Of the remaining students, only two students scored above the sixth stanine. This was on the concepts subtests.

TES training experience. The two TES groups (TES and TES) were given the responsibility for providing daily instruction for one unit of mathematics instruction entitled "Functions and Equations." The "Functions and Equations" unit introduced equations and word problems involving functions and equations. Equations with one and two variables were presented. Three weeks prior to teaching the unit to the sixth-grade pupils the two TES groups received instruction on how they were to teach the pupils to whom they had been assigned.

TES received specific instructions on how to perform the functions and associated tasks starred in Tables 1-4. Notice that Tables 1-4 correspond to Steps 6-8 shown in Figure 1.

Table 1

An Analysis Showing the Functions and Related Tasks for Step 6 of the Goal-Referenced Instructional Model—Diagnose Needs and Define Objectives

Functions		Related tasks	
*A.	Identifying needs of learners	*1.	Selecting and administering general placement tests or interviews
		*2.	Making gross profiles of each student's performance on subparts of placement tests or interviews
		*3.	Performing task analyses of items missed on placement tests or interviews
		*4.	Constructing or selecting pretests to determine which learning sequences should become instructional objectives
		*5.	Collecting data about students' interests and attitudes.
*B.	Defining operational objectives	*1.	Relating objectives to learners' diagnosed needs and interests
		*2.	Specifying instructional conditions
		*3.	Specifying observable student behaviors
		*4.	Selecting adequate performance criteria
*C.	Advising and guiding learners' attempts to identify objectives	*1.	Conducting individual conferences and interviews
		2.	Conducting small-group seminars
		3.	Structuring a decision-making procedure
		4.	Providing information that helps students identify their interests and needs

Table 2

An Analysis Showing the Functions and Related Tasks for Step 7 of the Goal-Referenced Instructional Model—Prepare an Instructional Plan

Functions		Related tasks	
*A.	Creating and constructing instructional units	*1.	Planning overall unit strategy based on analysis of learner needs
		2.	Preparing a content outline
		3.	Specifying unit goals, content-process objectives
		4.	Relating unit goals to types of individualized instruction
		*5.	Developing goal-referenced activities and curriculum materials
		6.	Developing file of resource persons, facilities, and media devices
		*7.	Generating analogous and equivalent practice exercises
*B.	Sequencing and organizing instructional objectives	1.	Selecting a sequence for unit goals by using Bloom's taxonomies
		*2.	Specifying entry, enroute, and terminal objectives for each unit goal
		*3.	Developing criterion tests for various instructional activities and experiences
*C.	Selecting and prescribing instruction	1.	Selecting type of individualized instruction based on learner's background
		*2.	Prescribing instructional activities based on diagnosed needs of learner
D.	Advising and guiding learners' attempts to select and sequence learning activities	1.	Structuring a decision-making procedure
		2.	Providing information regarding availability of instructional activities and materials
		3.	Conducting small-group seminars and individual conferences

Table 3
An Analysis Showing the Functions and Related Tasks for Step 8 of the Goal-Referenced Instructional Model—Implement the Instructional Plan

Functions		Related tasks	
*A.	Supporting and maintaining a learning resource center	*1.	Supervising use of self-instructional materials
		2.	Cataloging and cross-referencing instructional materials
		*3.	Circulating and accounting for the distribution of materials
		4.	Ordering, inventorying, and filing new materials
*B.	Facilitating student-teacher interaction	*1.	Creating and maintaining a supportive classroom climate
		*2.	Leading and focusing small-group discussions
		3.	Conducting large-group instruction
		4.	Providing feedback concerning the roles played by teachers and learners during group interaction
C.	Coordinating and supervising staff and student use of facilities and instructional materials	1.	Establishing schedules to coordinate staff and student use of instructional materials and facilities
		2.	Setting priorities for use of instructional materials and facilities

Table 4
An Analysis Showing the Functions and Related Tasks for Step 9 of the Goal-Referenced Instructional Model—Evaluate Learning and Instruction

Functions		Related tasks	
*A	Selecting and constructing tests, rating scales, and nontest procedures	1.	Constructing tables of specifications for test construction
		2.	Checking test validity
		3.	Checking test reliability
		*4.	Maintaining test file for diagnosis and post-instruction testing and evaluation
*B.	Judging and summarizing results of learning and instruction	*1.	Evaluating adequacy of instructional activities and materials
		2.	Evaluating adequacy of instructional objectives
		*3.	Evaluating adequacy of student attitude, effort, and performance
		*4.	Scoring and recording test and nontest data
*C.	Advising and guiding students' evaluation of their progress toward self-identified objectives	1.	Structuring a decision-making procedure
		*2.	Conducting individual and group evaluation sessions

Role of the Teacher

The tasks associated with the starred functions became the set of competencies used to instruct TES . These competencies are part of a set developed by the mathematics education faculty for use in its ongoing CBTE training program now in operation at Penn State. All other tasks were performed by the master teacher, the methods course instructor.

TES received instruction on how to do the following:

1. Use the suggestions in the teacher's guide for the assigned classroom text
2. Sequence instruction using a teacher's plan book format
3. Keep anecdotal records of the general progress of the students
4. Keep a student-teaching log of their experience

All other tasks were performed by the master teacher, the methods course instructor.

Both TES and TES were given the same number of class hours of instruction and review of the mathematics content to be taught. A set of behavioral teaching competencies was used with both groups to assist in evaluating the methods instructor's performance and to insure, as much as possible, that both TES and TES had successfully completed their training programs. The behavioral competencies and instructional sequences for both groups have been assembled to provide a detailed description of the instruction given each TES group (Fennell, 1972). A sampling of the TES behavioral competencies are presented below:

1.0 The students will know and comprehend why behavioral objectives are used and how to write them.

1.1 Given	Required Performance	Criteria
Heimer, Ralph T. Setting Goals for Mathematics Education. 1970.	Read and complete exercises.	80% mastery on exercises and oral discussion.

1.2 Given	Required Performance	Criteria
The investigator's lesson plan form	Construct a lesson plan using said form	100% mastery

1.3 Given	Required Performance	Criteria
Avital, S. M., and Shettleworth, S. J. Objectives for Mathematics Learning: Some Ideas for the Teacher. Bulletin No. 3. Toronto: The Ontario Institute for Studies in Education, 1968.	Read and comment	oral discussion and acknowledgement

2.0 The students will apply knowledge of diagnosis, sequencing of instruction, and individualization of instruction to various learning activities.

2.1 Given	Required Performance	Criteria
Trueblood, Cecil R. "A Model for Using Diagnosis in Individualizing Mathematics Instruction in the Elementary School Classroom" Arithmetic Teacher. Vol. 18, No. 7, November, 1971. pp. 505-513.	Read and react.	oral discussion and acknowledgement

Analysis

The formative data include: (a) pupils' pretest, posttest, and retention test scores on a 32-item unit mastery test; (b) pupils' pretest, posttest, and retention test scores on the Suydam-Trueblood Attitude toward Mathematics scale (1969); and (c) the amount of time needed by the instructional groups to demonstrate masery of the unit objectives. The unit mastery test, Forms A and B, was developed from a table of specifications based on the behavioral objectives for the unit of instruction. On a pilot administration of the unit mastery test, the test-retest reliability was .89. Item 18 of both forms appears below:

Form A

18. Tim bought 7 bags of 9 marbles.
He gave 6 marbles to Brett.
How many marbles did Tim have left?

A. 69 C. 33

B. 51 *D. 57

Form B

18. Joe had 7 groups of 8 comic books. He gave 9 comic books to Brad.
How many comic books did Joe have left?

A. 49 *C. 47

B. 57 D. 65

The Suydam-Trueblood attitude scale has an average internal consistency (Cronbach's Coefficient Alpha) of .96 (Bechtel, 1969).

Technically, the analysis of variance described below requires the use of independent data. In this case, the investigators were convinced that the large group differences observed on the unit mastery test were not due to violating the independence assumption of the analysis mode. Therefore, students, rather than class, were chosen as the unit of analysis, and a single classification ANOVA with repeated measures was computed for each instructional group to assess achievement gain. A Newman-Keuls analysis (Winer, 1962) was used to

locate significant differences. The single classification ANOVA and the Newman-Keuls analysis were also used with the Suydam-Trueblood Attitude toward Mathematics scale scores.

Results

Unit mastery test analysis. The single classification ANOVA for the scores on TES_1 pupils (N=20) indicated that these subjects made significant gains on the 32-item unit mastery test. See Table 5.

Table 5
Summary of the Results of a Repeated Measures ANOVA on Pretest, Posttest, and Retention Unit Mastery Test Administration for TES_1 Pupils, $N = 20$

Source	*SS*	*MS*	*df*	*F*
Treatment	1193.63	596.82	2	78.29*
Error	289.70	7.62	38	
Total	1483.33		40	
	Pretest	Posttest	Retention	
$\bar{X}$	17.90	26.10	28.65	
s	4.76	3.65	2.43	

*$p < .05$

The Newman-Keuls analysis showed that the significant gains were from pretest to retention test, pretest to posttest, and posttest to retention test. These findings indicated that TES_1 teachers were quite effective in their instruction, since the TES_1 pupils made significant gains in achievement.

A second single classification ANOVA was used to determine the gain made by TES_2 pupils (N=27) on the unit mastery test. Once again, significant differences were found. See Table 6.

Table 6
Summary of the Results of a Repeated Measures ANOVA on Pretest, Posttest, and Retention Unit Mastery Test Administration for TES_2 Pupils, $N = 27$

Source	*SS*	*MS*	*df*	*F*
Treatment	612.67	306.33	2	46.31*
Error	344.00	6.62	52	
Total	956.67		54	
	Pretest	Posttest	Retention	
$\bar{X}$	22.00	27.78	27.89	
s	4.60	3.02	3.31	

*$p < .05$

The Newman-Keuls test located the differences as being between pretest and retention test and between pretest and postest. However, there was no significant gain from posttest to retention test for the TES_2 pupils. These findings

indicate that TES_2 were also effective in their instruction, since the TES_2 pupils made significant gains in achievement.

Attitude scale analysis. A single classification ANOVA for the attitude scores of the TES_1 pupils produced a significant gain in attitude for these pupils. See Table 7.

Table 7
Summary of the Results of a Repeated Measures ANOVA on Pretest, Posttest, and Retention Attitude Scale Administration for TES_1 Pupils, $N = 16$**

Source	*SS*	*MS*	*df*	*F*
Treatment	780.88	390.44	2	5.94*
Error	2970.46	99.02	30	
Total	3751.34		32	
	Pretest	Posttest	Retention	
$\bar{X}$	79.13	86.38	88.56	
s	12.93	10.27	14.35	

* $p < .05$
** Note the *N*s in Tables 5 and 7 differ because four subjects missed at least one attitude scale administration.

The Newman-Keuls test indicated that the gain from pretest to posttest and from posttest to retention test administrations of the Suydam-Trueblood Attitude toward Mathematics scale was not significant for the TES_1 pupils. However, there was a significant gain from pretest to retention test. These findings indicated that the TES_1 type of individualized instruction seemed to appeal to the TES_1 pupils involved.

The single classification ANOVA for TES_2 pupils' attitude scores was not significant. See Table 8.

Table 8
Summary of the Results of a Repeated Measures ANOVA on Pretest, Posttest, and Retention Attitude Scale Administration for TES_2 Pupils, $N = 24$*

Source	*SS*	*MS*	*df*	*F*
Treatment	10.53	5.26	2	.05
Error	4486.14	97.52	46	
Total	4496.67		48	
	Pretest	Posttest	Retention	
$\bar{X}$	86.88	85.96	86.25	
s	19.26	20.86	17.44	

* Note the *N*s in Tables 6 and 8 differ because three subjects missed at least one attitude scale administration.

This indicated that TES_2 pupils did not make significant changes in attitude scores as a result of their instruction.

Table 9 shows the wide differences in the amount of time needed by TES_1 and TES_2 pupils to complete the unit of mathematics content.

Table 9
A Frequency Distribution Comparing the Time Used by TES_1 and TES_2 to Complete a Unit of Instruction on Functions

Number of 45-minute instructional periods	Number of pupils	
	TES_1 (N = 20)	TES_2 (N = 27)
10	4	0
11	0	0
12	0	0
13	1	0
14	2	0
15	1	0
16	2	0
17	7	0
18	1	0
19	2	27

All TES_2 pupils who followed the textbook sequence took 19 45-minute instructional periods to complete the unit. In contrast, half of the TES_1 pupils completed their work in 16 instructional periods or less. One-fifth finished the unit in 10 periods.

Discussion

The focus of this report has been to present formative data to show the impact that two field-based teacher-training experiences had on low-achieving sixth-grade pupils. Both TES groups significantly increased their pupils' unit mastery test scores without negatively affecting their attitude toward elementary school mathematics. Therefore, using the elementary school as a training laboratory for the TES in this study seemed to have positive impact on the pupils they taught.

The data from this study suggest certain ideas that have potential application to the field-based preparation of prospective elementary teachers:

1. Having elementary teacher education students help teach a diagnostically prescribed unit of mathematics under the supervision of a master teacher may be educationally defensible in terms of the positive impact it can have on the pupils and the realistic training experience it can afford the prospective teachers.

2. Public schools and teacher education institutions could consider combining forces to assist those pupils who most need clinical attention - the low-achieving pupils.

Future study and formative research currently in progress will indicate more popular and most essential components within the diagnostic-prescriptive mathematics methods orientation. It would also be advisable to consider the proposed teacher education program as a graduate level or teacher inservice

offering for retraining teachers or updating clinicians and/or teacher aides working in a diagnostic-prescriptive mathematics setting.

As we continue to invest in providing for diagnostic-prescriptive strategies in mathematics there will be a need to continually focus attention on the prospective teacher and his/her readiness for the challenges of diagnostic-prescriptive mathematics instruction. Preservice diagnostic-prescriptive experiences within the mathematics methods class have proven to be a valuable and essential component of elementary teacher training at the undergraduate level.

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